The Case for Tax-Adjusted Inflation Targeting

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Abstract

How should monetary policy react to inflation rate increases that result from indirect tax increases? We provide evidence that in the real world central banks fight against these spikes in headline inflation, fearing that the spike will lead to further wage and price inflation. Introducing indirect taxes into a standard New-Keynesian model we show that the optimal monetary policy is to keep real interest rates unchanged and target tax-adjusted inflation instead. This result follows directly from assuming that prices reflect the indirect tax increases immediately, but that they are otherwise sticky. We conclude that tax-adjusted inflation targeting is an easy way to improve monetary policy in practice. Further, we show that the switch to tax-adjusted inflation targeting affects the impact of indirect tax increases on GDP, with implications for fiscal multipliers.

1 Introduction

How should central banks react to the inflation spikes that result from indirect tax increases? Central banks fight against these inflation rate increases by raising interest rates. But should they do otherwise? We argue that central banks should target tax-adjusted inflation, and not headline inflation. In other words, we argue that central banks should allow the inflation rate spikes that result from indirect tax increases - rather than fighting against them.

Our evidence that central banks respond to the spikes in headline inflation that follow indirect tax increases is based on three sources: (i) our study of the reactions of the Reserve Bank of Australia and the Bank of England, (ii) cross-country evidence provided by the International Monetary Fund (IMF, 2010), and (iii) comments by the central bank governors of Australia and the United Kingdom that this reaction was their intention. All this evidence shows that central banks fight against the inflation that results from indirect tax increases.

To characterize the optimal response to the inflation spikes caused by indirect taxes we introduce indirect taxes into a basic new Keynesian model with sticky prices. We find that optimal monetary policy is to allow these spikes. To obtain this result, we assume that prices respond immediately to changes in indirect taxation, even though they are otherwise sticky.

We contend that our assumption that prices respond immediately to indirect tax increases is reasonable. When indirect taxes increase, most prices such as restaurant menus, electricity bills, and supermarket receipts are all adjusted. Therefore the usual arguments for price stickiness based on menu costs do not apply. The same logic applies to the rational inattention argument for sticky prices. Indirect tax increases are always widely publicized, discussed on TV, and appear in tax returns. All of which makes indirect tax increases very hard to ignore. So both of the standard theoretical justifications for sticky prices suggest that prices will respond immediately to indirect tax increases.

The standard objection to allowing the spikes in inflation when indirect taxes increase is second round effects: that a spike in price inflation will cause workers to demand higher wages, and then firms will increase prices again, setting off a price-wage spiral. We extend to a standard New-Keynesian model with sticky wages to allow for this objection. Tax-adjusted inflation targeting remains optimal taking into account the second round effects that arise from the pass-through of the inflation spike into inflation rate expectations and wage inflation. Some wage inflation does follow the headline inflation spike as workers recover their lost purchasing power. However this does not lead to an inflation outbreak. Firms are already facing lower demand because of the tax increase, so they avoid reacting to the wage inflation by raising prices as this would further harm demand. Firms prefer instead to lower output as real wages return to their earlier levels. Relatedly, at the aggregate level an indirect tax increase reduces the efficient level of output.

Is this issue more than a mere technicality? We think so. Tax-adjusted inflation targeting is derived in the models as the welfare-maximizing policy. The change from targeting headline inflation to tax-adjusted inflation also has implications for the effects of fiscal stimulus and austerity on GDP. The effects of different types of austerity, tax increases vs spending cuts, on GDP is an open question in economics (Alesina and Ardagna, 2010; Alesina and Giavazzi, 2012; Alesina, Favero, and Giavazzi, 2012). The International Monetary Fund (2010) find that most of the difference in the effects of tax increases vs spending cuts on current GDP comes from the differences in the reactions of monetary policy and, relatedly, exchange rates. So if the monetary policy currently pursued, targeting headline inflation, is the 'wrong' one, then switching to the optimal policy of targeting tax-adjusted inflation has implications in terms of the effects of different types of austerity on GDP. DeLong and Summers (2012) go as far as to comment that "the most important issue in thinking about the fiscal multiplier is the response of monetary policy to fiscal policy". We show that switching to tax-adjusted inflation targeting reduces the fall in GDP following indirect tax increases by 1% of GDP.

2 What Central Banks Do

Evidence shows that central banks increase interest rates to fight against the increase in inflation caused by tax increases. While central banks recognize that the increase in inflation is due to taxes, they nonetheless tighten monetary policy on the grounds that such a temporary increase might lead to higher inflation expectations. Central banks currently target headline inflation and not tax-adjusted inflation[1]. In fact, many central banks explicitly declare inflation targeting as a policy objective.

We begin with the case of Australia where on July 1st 2000 a 10% indirect tax (the Goods and Services Tax) was introduced. The resulting spike in inflation clearly stands out from trend

[1] Most central banks are better characterized as targeting core inflation – excluding prices of goods such as food, fuels, & sometimes commodities. However this distinction between core and headline inflation is peripheral to the issue of indirect taxes and so is left aside to avoid complicating the issue unnecessarily. Two central banks, Canada & New Zealand, do in fact partially tax-adjust inflation but this is the exception rather than the rule (Bernanke and Mishkin, 1997).

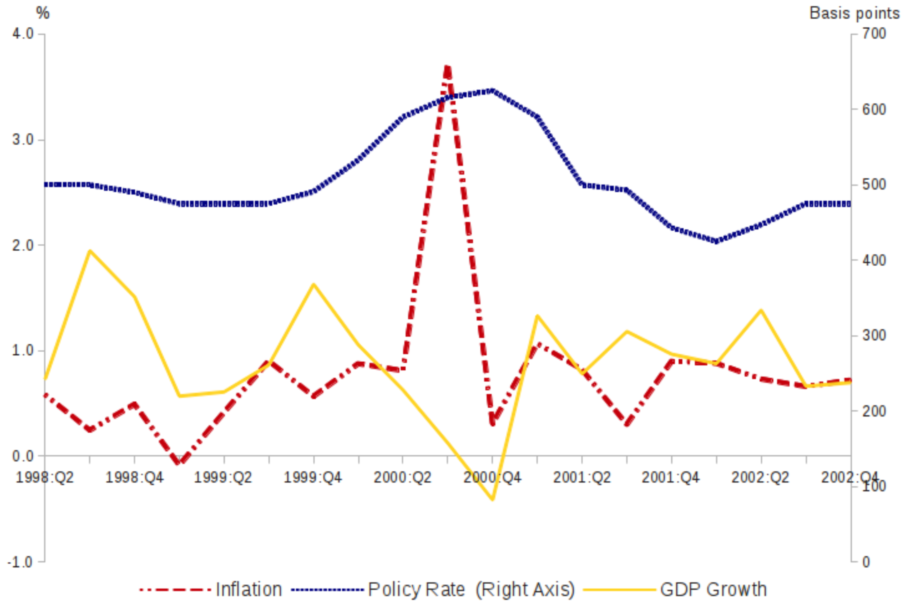


Figure 1: Australia: Introduction of 10% GST

inflation, as seen in Figure 1. Statements by then Governor of the Reserve Bank of Australia Ian McFarlane (2000b; 2000a), mention the movements in inflation, but include no discussion of whether they are due to the tax increase (the possible roles of energy prices and international factors in inflation are considered). This episode clearly shows GDP growth falling while the one-off spike in inflation is due to the tax increase. The increase in interest rates is a reaction to this spike. The downward movements of GDP ruling out the possibility that the bank was reacting to an overheating economy (employment, not shown, displays much the same behaviour as GDP growth).

The IMF (2010) provides further evidence. Their data consists of a sample of 32 fiscal adjustments (large changes in either taxes or government spending) in the advanced economies over the past 30 years. Applying panel data methods they provide, among other things, evidence on the reactions of monetary policy to these fiscal adjustments. What is clear from their work is that increases in taxes, and especially increases in indirect taxes, are met with an increase in policy interest rates by central banks. This is seen in the impulse response functions for the reaction of interest rates to indirect taxes shown here in Figure 2[2]. This confirms analytically what we saw in the Australian experience: central banks fight against the spikes in inflation caused by indirect tax increases, raising policy interest rates.

Lastly we analyze the United Kingdom (UK). The UK government, after having a Value Added Tax (VAT) rate of 17.5% since 1991 has recently changed rates a number of times. First reducing VAT to 15% in December 2008, returning to 17.5% in January 2010, and finally increasing further to 20% in January 2011. We perform an econometric analysis of the reaction of the Bank of England to these tax changes by estimating a Taylor rule for interest rates. Again we find that the central bank increases interest rates in response to inflation resulting from consumption tax increases. The details of this are left till later in the paper as it first requires the development of some theory to interpret the results. For now we content ourselves with a quotation from the minutes of meetings

[2]Forms part of Figure 3.7 of International Monetary Fund (2010)

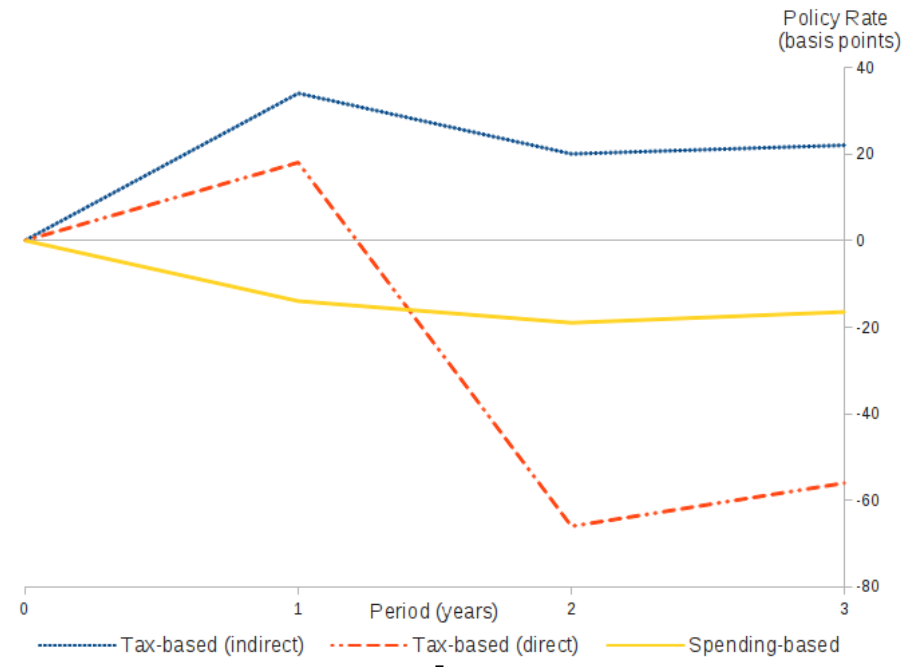


Figure 2: Impact of a 1% of GDP Fiscal Consolidation on Interest Rates (International Monetary Fund)

of the Bank of England (2011a) just after the January 2011 increase in VAT stating that they were aware that *'Inflation had been boosted by the ... increases in VAT'* and that this currently higher inflation was *'likely to exacerbate the risk that expectations of above-target inflation would become ingrained, affecting wage and price pressures'*.

3 What Central Banks Should Do

A well-known prescription of the basic New-Keynesian model is that optimal monetary policy is inflation targeting. Not because of some inherent desirability of inflation targeting, but as a way to maximize welfare by achieving the efficient level of output - avoiding the distortions arising from sticky prices. But what should central banks do in reaction to inflation spikes arising from indirect tax increases? When prices immediately reflect increases in indirect taxes but are otherwise sticky, maximizing welfare is still achieved by the efficient level of output. But this is no longer achieved by targeting inflation. Instead targeting tax-adjusted inflation is the optimal monetary policy[3].

The assumption that prices immediately reflect indirect tax increases follows from the usual arguments justifying sticky prices. There are two standard theoretical arguments for price-stickiness: the existence of menu costs, and rational inattention on the part of price setters. We address these in turn.

Adapting prices to changing market conditions is not costless. New menus must be printed and advertisements updated. These menu costs cause firms to avoid constantly changing prices. However given that tax changes require businesses to change their accounting such menu costs are

[3]This result is not unlike that of Aoki (2001) who shows that if non-core prices are not-sticky then optimal policy is to target core inflation, rather than headline inflation.

being incurred anyway. The case of indirect taxes is especially stark as the tax changes must be reflected in the receipts the business issues which are often required by law to tell the customer how much of the bill is attributable to indirect taxes. Since firms are incurring these menu costs anyway, they would be foolish indeed not to change their prices while they are at it. Thus, the menu costs justification for sticky prices suggests prices should react immediately to tax changes.

Rational inattention argues that prices are sticky because adjusting them constantly would require firm owners to pay attention to everything that goes on. Since people have a limited amount of time it is not possible to pay attention to everything, and firm owners should direct their limited attention to those things that are more important to their business such as developing new products and attracting customers. Fluctuations in prices and interest rates are not among the more important things for their profitability and therefore it is rational to give them less attention, leading prices to be sticky in relation to these fluctuations[4]. So the rational inattention argument also suggests that generally sticky prices will nonetheless immediately adjust to tax changes. Tax changes certainly get the attention of business owners having, in addition to their implications for profitability, various legal implications for anyone running a business, and so they will adjust their prices to taxes. To put it bluntly, ignoring taxes is unlikely to be rational!

Unfortunately we are not able to directly test our assumption that prices immediately reflect changes in indirect taxes. Empirical support for price stickiness comes from papers such as Nakamura and Steinsson (2008) which measure the frequency of price changes using large data sets on prices at the level of individual goods. With such a database covering a period in which indirect taxes are changed the assumption could be directly tested: the assumption predicts a much larger number of price changes than usual when the tax change occours. The case of Australia seen earlier, where the price spike occours simultaneously with the indirect tax increase suggests that the assumption is reasonable.

3.1 Basic New Keynesian Model with Indirect Taxes

We start by extending the basic New Keynesian model[5]. In this model sticky prices are modeled a la Calvo (1983). We add indirect taxes to this model. We then model pre-tax prices as being sticky a la Calvo. After-tax prices are just the sticky pre-tax price plus the current tax rate[6]. So prices are sticky, but immediately reflect changes to indirect taxes. Consumers care only about after-tax prices, while firms care about pre-tax prices. In this basic model we derive the analytical result that optimal monetary policy is to target tax-adjusted inflation.

We describe the micro-foundations of the model and then give the system of equations derived from these which describe the dynamic behaviour of the system. The full derivation of the system of equations from the micro-foundations can be found in Appendix B. The sufficient conditions for optimal monetary policy are then given and their implication of targeting tax-adjusted inflation is derived. Taxes are denoted by $\mathcal{T}$ and assumed to follow a stationary stochastic process (say eg. AR(1)). Tax revenue is simply returned as a lump-sum transfer, as this allows us to concentrate directly on the effect of tax changes on inflation without worrying about the effect of government spending on inflation, or the use of government debt which may later be monetized. Lower-case

[4] See, eg. Mackowiak and Wiederholt (2009).

[5] Specifically, that of Chapter 4 of Galí (2008).

[6] This assumption that the entire tax increase is passed directly into consumer prices is an approximation. Intuitively the actual amount that would pass into consumer prices would reflect the relative tax incidence of indirect taxes on consumers and firms. However, this does not affect the intuition of the model's policy prescriptions.

letters are used throughout to denote the log-deviations from steady-state of the corresponding upper-case letter.

3.1.1 Households

There are a continuum of goods indexed by $i \in [0,1]$. Let $P_t(i)$ be the pre-tax price of good i, so final prices are given by $(1+\mathcal{T}_t)P_t(i)$. A representative agent maximizes his expected discounted utility choosing hours worked, consumption, and savings. Consumption is given by a constant elasticity of substitution index $C_t = \left(\int_0^1 C_t(i)^{\frac{\epsilon-1}{\epsilon}} di\right)^{\frac{\epsilon}{\epsilon-1}}$, where $C_t(i)$ is consumption of differentiated good i. This maximization is done subject to the budget constraint $\int_0^1 (1+\mathcal{T}_t)P_t(i)C_t(i)di + Q_t B_t \leq B_{t-1} + W_t N_t + T_t$, where B_t are purchases of bonds of price Q_t, W_t is the wage, N_t is hours worked, and T_t is a lump-sum transfer. The period utility function is given by $U(C_t, N_t) = \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi}$.

3.1.2 Firms

The firms problem is the same as that which occours in the absence of a consumption tax. Each consumption good is produced by a different firm, all of which have access to the same technology function, given by $Y_t(i) = A_t N_t(i)^{1-\alpha}$, where $Y_t(i)$ is output of good i, A_t is the technology level which is common across firms, and $N_t(i)$ is the labour employed by firm i. Each period firms are allowed to change prices with probability $1-\theta$. Thus the problem faced by a firm that reoptimizes it price in period t is to maximize its expected profits during the time in which this price, P_t^*, is expected to be in place,

$$\max_{P_t^*} \sum_{k=0}^{\infty} \theta^k E_t\{Q_{t,t+k}(P_t^* Y_{t+k|t} - \Psi_{t+k}(Y_{t+k|t}))\}$$

Subject to a demand function that is derived from the first-order conditions of the consumers problem, namely

$$Y_{t+k|t} = \left(\frac{P_t^*}{P_{t+k}}\right)^{-\epsilon} C_{t+k}$$

where $\Psi_{t+k}(\cdot)$ is the cost function, $Q_{t,t+k}$ is the stochastic discount factor for nominal payoffs, $Y_{t+k|t}$ is the production in period $t+k$ of a firm that last reset it price in period t, and $P_t = [\int_0^1 P_t(i)^{1-\epsilon} di]^{\frac{1}{1-\epsilon}}$ is the aggregate price level.

3.1.3 Price Inflation Dynamics

The evolution of the aggregate consumer price level (an index of the after-tax prices for the individual goods) is given by

$$(1+\mathcal{T}_t)P_t = \left[\theta\left(\frac{1+\mathcal{T}_t}{1+\mathcal{T}_{t-1}}P_{t-1}\right)^{1-\epsilon} + (1-\theta)((1+\mathcal{T}_t)P_t^*)^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}}$$

thus consumer price inflation is

$$\Pi_t^{1-\epsilon} = \theta\left(\frac{1+\mathcal{T}_t}{1+\mathcal{T}_{t-1}}\right)^{1-\epsilon} + (1-\theta)\left(\frac{1+\mathcal{T}_t}{1+\mathcal{T}_{t-1}}\right)^{1-\epsilon}\left(\frac{P_t^*}{P_{t-1}}\right)^{1-\epsilon}$$

where $\Pi_t = \frac{(1+\mathcal{T}_t)P_t}{(1+\mathcal{T}_{t-1})P_{t-1}}$ is the consumer price inflation rate, and P_t is the pre-tax price. Note that inflation is thus a combination of changing taxes on the fraction prices that were not updated (the first term) plus changing after-tax prices for the fraction of prices that were updated.

3.1.4 Equilibrium

Market clearing in the model involves market clearing for each of the consumption goods, $C_t(i) = Y_t(i)$, $\forall\ i \in [0,1]$, $\forall\ t$, and in the labour market $N_t = \int_0^1 N_t(i)di$.

3.1.5 System of Equations

From these micro-foundations are derived the system of equations describing the behaviour of the model: the New Keynesian Phillips curve (NKPC) and the dynamic IS equation. See Appendix B for the full derivation of the system of equations from the micro-foundations. The NKPC is

$$\pi_t = \beta E_t\{\pi_{t+1} - \Delta\tau_{t+1}\} + \kappa\widetilde{y}_t + \Delta\tau_t$$

where $\kappa = \lambda(\sigma + \frac{\varphi+\alpha}{1-\alpha})$, $\lambda = \frac{(1-\theta)(1-\beta\theta)}{\theta}\Theta$, $\Theta = \frac{1-\alpha}{1-\alpha+\alpha\epsilon}$. The dynamic IS equation is

$$\widetilde{y}_t = -\frac{1}{\sigma}(i_t - E_t\{\pi_{t+1} - \Delta\tau_{t+1}\} - r_t^n) + E_t\{\widetilde{y}_{t+1}\}$$

where r_t^n is the natural rate of real interest, given by

$$r_t^n = \rho + \sigma E_t\{\Delta y_{t+1}^n\} = \rho + \sigma\psi_{ya}^n E_t\{\Delta a_{t+1}\} + \sigma\psi_{y\tau}^n \Delta\tau_{t+1}$$

where $\psi_{ya}^n \equiv \frac{1+\varphi}{\sigma(1-\alpha)+\varphi+\alpha}$ and $\psi_{y\tau}^n \equiv \frac{1-\alpha}{\sigma(1-\alpha)+\varphi+\alpha}$. τ_t denotes the log deviation from steady state of indirect taxes $1+\mathcal{T}_t$, $\widetilde{y}_t$ is the output gap (the difference between actual output y_t and the natural level y_t^n which would result under flexible prices), r_t^n is the natural interest rate (that associated with the flexible price output y_t^n), i_t is the nominal interest rate ($= -logQ_t$), and ρ is the discount rate ($= -log\beta$). We observe that the addition of taxes alters the natural level of output and the natural rate of interest, both now depend on the taxes. Together with a monetary policy rule defining the evolution of i_t these equations form a system of equations that fully describe the evolution of the model.

3.1.6 Optimal Monetary Policy

When considering optimal monetary policy one further assumption is required. Following the literature, it is assumed that the distortion caused by the market power of the firms arising from monopolistic competition is not something to be considered by monetary authorities. For this reason a wage-subsidy is assumed that makes the equilibrium under flexible prices efficient. With this wage-subsidy in place the decentralized equilibrium is efficient, corresponding to that which would be chosen by as social planner. For our purposes, the wage-subsidy is also assumed to balance the distortions of the consumption tax to avoid monetary policy trying to fight this. Monetary policy aims to avoid distortions arising from sticky-prices, both from the average marginal costs diverging from their optimal level, and from distortions in relative prices. Thus, optimal policy will be that which keeps the output gap closed.

Assuming that there are no initial relative distortions in prices ($P_{-1}(i) = P_{-1}, \forall\ i \in [0, 1]$), we have that optimal monetary policy will be that which closes the output gap ($\widetilde{y}_t = 0, \forall\ t$). From the NKPC we thus have that optimal policy is characterized by

$$\pi_t = \Delta\tau_t$$

So optimal policy is to target the tax-adjusted inflation rate (given by $\Pi_t^{adjusted} = \frac{P_t}{P_{t-1}}$). Targeting headline inflation is suboptimal. This is in contrast to the standard model where optimal policy is to target inflation.

The dynamic IS equation implies that this can be done using the monetary policy rule $i_t = r_t^n$, ie. setting the nominal interest rate equal to the natural real interest rate[7]. Note that the natural real interest rate depends on the current tax rate.

4 What About Second-Round Effects?

The main objection to allowing inflation spikes is that doing so will set off a price-wage inflation spiral. Seeing the spike, workers demand increased wages setting off further price increases by firms, embedding inflation into expectations and starting an inflation spiral. Second-round effects refer to these further price increases and the resulting price-wage inflation spiral. In the words of the Bank of England (2011a): *'[R]eports from the Banks Agents suggested that it was also possible that the pass-through into consumer prices of Januarys VAT increase would be greater than previously expected. These factors [the VAT and price inflation in imports]... were also likely to exacerbate the risk that expectations of above-target inflation would become ingrained, affecting wage and price pressures.'*

To address this objection sticky wages are now added to the model. This allows both for the risk of changes in inflation expectations, and for inflation to become embedded in wages. In the standard new Keynesian model (without taxes), the addition of sticky wages modifies the optimal monetary policy. Instead of targeting price inflation it is instead optimal to target a weighted combination of price & wage inflation. Introducing indirect taxation, optimal monetary policy becomes to target a weighted combination of tax-adjusted price & wage inflation.

In addition to the inflation spike, an indirect tax increase causes a drop in demand. Facing low demand a firm will avoid increasing prices as this would lead to even lower demand for its product. Instead it decreases production toward the new lower efficient level of output implied by the lower demand for it's product. At first this drop in demand is partially offset by a fall in real wages: nominal wages are unchanged while prices jump, so at first firms only partially decrease output. As workers recover the purchasing power of their wages, real wages increase, resulting in some wage inflation. Still facing low demand for their products, firms prefer to decrease output than lower demand further by increasing prices. This aversion to further harming demand by increasing prices explains why optimal monetary policy is not changed by the possibility of second-round effects. Optimal monetary policy allows the inflation spike to occour while keeping real interest rates unchanged; obviously this leads to a momentary jump in the nominal interest rate – which equals the real interest rate plus inflation – when the inflation spike occours, but which is otherwise

[7] There is an issue of uniqueness of the equilibrium, which can be resolved with a slightly different rule for the nominal interest rate (see Galí (2008)). However this is peripheral to our interest here in the the characterization of the optimal policy in terms of inflation.

unchanged. By trying to fight the spike current monetary policy causes an unnecessarily large fall in output; an issue we address in Section 5.

4.1 Standard New Keynesian Model with Indirect Taxes

We extend the standard New Keynesian model of sticky prices and sticky wages to incorporate indirect taxes. Our treatment is based on Galí (2008) Chapter 6, which in turn introduces sticky wages following Erceg, Henderson, and Levin (2000). As before, when introducing the taxes we model the pre-tax prices as sticky, with taxes added on top of these. This captures our assumption that prices, while sticky, immediately reflect indirect tax increases. A description of the micro-foundations and the resulting system of equations follows, again for a full derivation one is referred to the Appendix C. We begin by looking at the firms problem.

4.1.1 Firms

As in our treatment of the basic sticky prices model, a continuum of firms is assumed, indexed by $i \in [0, 1]$, each of which produces a differentiated good with a technology represented by the production function $Y_t(i) = A_t N_t(i)^{1-\alpha}$, where $Y_t(i)$ denotes the output of good i, A_t is an exogenous technology parameter common to all firms, and $N_t(i)$ is an index of labour input used by firm i and defined by

$$N_t(i) \equiv \left[\int_0^1 N_t(i,j)^{1-1/\epsilon_w} dj \right]^{\frac{\epsilon_w}{\epsilon_w - 1}}$$

where $N_t(i,j)$ denotes the quantity of type-j labour employed by firm i in period t. The parameter ϵ_w represents the elasticity of substitution among labour varieties. We assume a continuum of labour types, indexed by $j \in [0, 1]$.

Let $W_t(j)$ denote the wage for type-j labour in period t, for all $j \in [0, 1]$. Wages are set by workers. Given wages at time t for the different types of labour services, cost minimization yields a corresponding set of demand schedules for each firm i and labour type j, given the firm's total employment $N_t(i)$

$$N_t(i,j) = \left(\frac{W_t(j)}{W_t} \right)^{-\epsilon_w} N_t(i)$$

for all $i, j \in [0, 1]$, where $W_t \equiv \left[\int_0^1 W_t(j)^{1-\epsilon_w} dj \right]^{\frac{1}{1-\epsilon_w}}$ is an aggregate wage index.

Hence, and conditional on an optimal allocation of the wage bill among the different types of labour, a firm adjusting it's price in period t will solve the following problem, which is identical to the one analyzed in the standard model with sticky prices

$$\max_{P_t^*} \sum_{k=0}^{\infty} \theta_p^k E_t \{ Q_{t,t+k} (P_t^* Y_{t+k|t} - \Phi_{t+k}(Y_{t+k|t})) \}$$

subject to the sequence of demand constraints

$$Y_{t+k|t} = \left(\frac{P_t^*}{P_{t+k}} \right)^{-\epsilon_p} C_{t+k}$$

for $k = 0, 1, 2, ...,$ where notation is as before.

4.1.2 Households

To introduce sticky-wages we have assumed that each household supplies a differentiated labour type indexed by $j \in [0,1]$. These are then aggregated into a single labour input used in production via a Dixit-Stiglitz aggregator. Every period with probability $1-\theta_w$ the household gets to set a new wage, otherwise it is stuck with the wage it had last period. The problem of a household that gets to set it's wage in period t thus becomes to maximize

$$E_t\left\{\sum_{k=0}^{\infty}(\beta\theta_w)^k U(C_{t+k|t}, N_{t+k|t})\right\}$$

subject to the sequence of labour demand schedules and flow budget constraints that are effective while W_t^* remains in place, ie.

$$N_{t+k|t} = \left(\frac{W_t^*}{W_t+k}\right)^{-\epsilon_w} N_{t+k}$$
$$(1+\mathcal{T}_{t+k})P_{t+k}C_{t+k|t} + E_{t+k}\{Q_{t+k,t+k-1}D_{t+k+1|t}\} \leq D_{t+k|t} + W_t^* N_{t+k|t} - T_{t+k}$$

for $k = 0,1,2,....$ Where $C_{t+k|t}$, $N_{t+k|t}$, & $D_{t+k|t}$ are consumption choice, labour supply choice, and portfolio of securities held in $t+k$ by households that last reset their wage in period t; all other notation as before.

We use the same utility function as previously, namely $U(C,N) = \frac{C^{1-\sigma}}{1-\sigma} - \frac{N^{1+\varphi}}{1+\varphi}$.

4.1.3 Wage Inflation Dynamics

Given the assumed wage setting structure, the evolution of the aggregate wage index is given by

$$W_t = [\theta_w W_{t-1}^{1-\epsilon_w} + (1-\theta_w)(W_t^*)^{1-\epsilon_w}]^{\frac{1}{1-\epsilon_w}}$$

4.1.4 Equilibrium

Goods market clearance is given by $Y_t(i) = C_t(i)$, $\forall i \in [0,1]$. The output gap is once more defined as $\tilde{y}_t \equiv y_t - y_t^n$, although the natural level of output, y_t^n, is now that which would occour in the absence of both price and wage stickiness. A new variable, the *real wage gap*, is defined as $\tilde{\omega}_t \equiv \omega_t - \omega_t^n$, where $\omega_t \equiv w_t - p_t - \tau_t$, denotes the real wage, and where ω_t^n is the natural real wage, the real wage that would prevail in the absence of nominal rigidities, and which is given by

$$\omega_t^n = log(1-\alpha) + \psi_{\omega a}^n a_t - \psi_{\omega\tau}^n \tau_t - \mu^p$$

where $\psi_{\omega a}^n \equiv \frac{1-\alpha\psi_{ya}^n}{1-\alpha} \geq 0$ and $\psi_{\omega\tau}^n \equiv \frac{1-\alpha\psi_{y\tau}^n}{1-\alpha} \geq 0$. ψ_{ya}^n and $\psi_{y\tau}^n$ are unchanged from the case without sticky wages; they determine the efficient level of output, which by definition is output when prices and wages are flexible.

4.1.5 System of Equations

From these micro-foundations, we derive the system of equations characterizing the dynamic behaviour of the model, the full derivation can be found in Appendix C. The first equation is the

New Keynesian Phillips Curve (NKPC)

$$\pi_t^p = \beta E_t\{\pi_{t+1}^p - \Delta\tau_{t+1}\} + \kappa_p \tilde{y}_t + \lambda_p \tilde{\omega}_t + \lambda_p \hat{\tau}_t + \Delta\tau_t \tag{1}$$

where $\kappa_p = \frac{\alpha\lambda_p}{1-\alpha}$ and $\lambda_p = \frac{(1-\theta_p)(1-\beta\theta_p)}{\theta_p}\frac{1-\alpha}{1-\alpha+\alpha\epsilon_p}$ Notice that $\omega_t \equiv w_t - p_t$, and p_t reacts to τ_t but w_t doesn't, hence ω_t does; this is why NKPC for prices now has the $\lambda_p\tilde{\omega}_t + \lambda_p\hat{\tau}_t$ term, which with flexible wages would be zero. With the introduction of sticky wages there is now also a NKPC for wages

$$\pi_t^w = \beta E_t\{\pi_{t+1}^w\} + \kappa_w \tilde{y}_t - \lambda_w \tilde{\omega}_t \tag{2}$$

where $\kappa_w = \lambda_w(\sigma + \frac{\varphi}{1-\alpha})$ and $\lambda_w = \frac{(1-\theta_w)(1-\beta\theta_w)}{\theta_w(1+\epsilon_w\varphi)}$. In addition, there is an identity relating the changes in the wage gap to price inflation , wage inflation, and the natural wage

$$\tilde{\omega}_t \equiv \tilde{\omega}_{t-1} + \pi_t^w - \pi_t^p - \Delta\omega_t^p \tag{3}$$

we once again get the dynamic IS equation

$$\tilde{y}_t = E_t\{\tilde{y}_{t+1}\} - \frac{1}{\sigma}(i_t - E_t\{\pi_{t+1}^p\} - r_t^n)$$

where, as in case without sticky wages

$$r_t^n = \rho - \sigma E_t\{\Delta y_{t+1}^n\} = \rho - \sigma\psi_{ya}^n E_t\{\Delta a_{t+1}\} + \sigma\psi_{y\tau}^n E_t\{\Delta\tau_{t+1}\}$$

however this should now be understood as the rate prevailing in an equilibrium with both flexible wages and prices. Closing, the model requires the choice of the interest rate i.

4.2 Behaviour under the Optimal Monetary Policy

Define optimal monetary policy to be that which maximizes welfare. It can be shown[8] that, based on an approximation of the utility function, the welfare expressed as a fraction of steady state consumption is given by

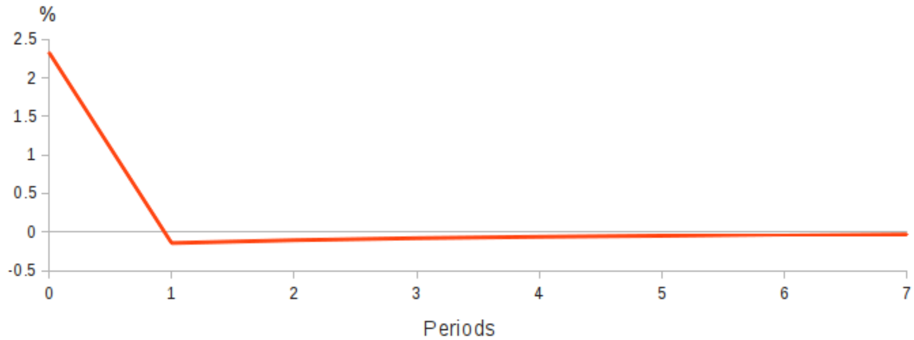
$$\mathbb{W} = \frac{1}{2}E_0\sum_{t=0}^{\infty}\beta^t\left((\sigma + \frac{\varphi+\alpha}{1-\alpha})\tilde{y}_t^2 + \frac{\epsilon_p}{\lambda_p}(\pi_t^p)^2 + \frac{\epsilon_w(1-\alpha)}{\lambda_w}(\pi_t^w)^2\right) + t.i.p \tag{4}$$

where $t.i.p.$ collects various terms that are independent of policy. Ignoring the latter terms we can express the average period welfare loss as

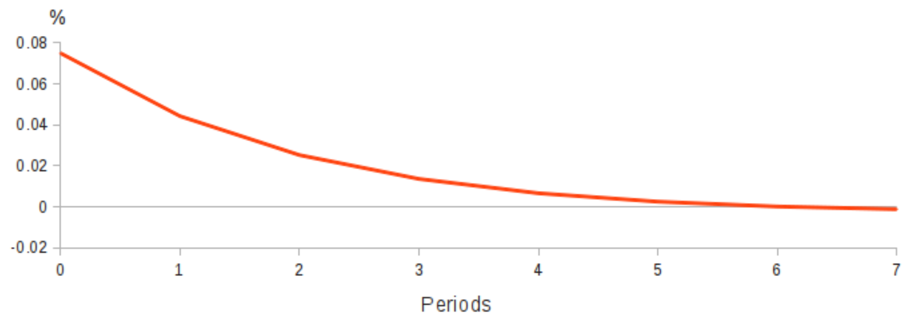
$$\mathbb{L} = (\sigma + \frac{\varphi+\alpha}{1-\alpha})var(\tilde{y}_t) + \frac{\epsilon_p}{\lambda_p}var(\pi_t^p) + \frac{\epsilon_w(1-\alpha)}{\lambda_w}var(\pi_t^w) \tag{5}$$

We now take a primal approach to characterizing optimal monetary policy, that is, we characterize the behaviour of the economy under the optimal policy without actually calculating what form it takes as an interest rate rule. Optimal monetary policy is given by the central bank seeking to maximize welfare, (4), subject to the system of equations describing the economy, (1), (2) & (3)

[8]See Galí (2008) Appendix 6.2; the proof carries over directly to the case with consumption taxes and sticky pre-tax prices



(a) Price Inflation



(b) Wage Inflation

Figure 3: IRFs to an increase in indirect taxes of 2.5% under the optimal monetary policy

for $t = 0, 1, 2, ...$ Let $\{\xi_{1,t}\}$, $\{\xi_{2,t}\}$, & $\{\xi_{3,t}\}$ denote the sequence of Lagrange multipliers associated with these constraints. The optimality conditions for the optimal policy are thus given by

$$\left(\sigma + \frac{\varphi + \alpha}{1 - \alpha}\right) \tilde{y}_t + \kappa_p \xi_{1,t} + \kappa_w \xi_{2,t} = 0 \tag{6}$$

$$\frac{\epsilon_p}{\lambda_p} \pi_t^p - \Delta \xi_{1,t} + \xi_{3,t} = 0 \tag{7}$$

$$\frac{\epsilon_w (1 - \alpha)}{\lambda_w} \pi_t^w - \Delta \xi_{2,t} - \xi_{3,t} = 0 \tag{8}$$

$$\lambda_p \xi_{1,t} - \lambda_w \xi_{2,t} + \xi_{3,t} - \beta E_t\{\xi_{3,t+1}\} = 0 \tag{9}$$

for $t = 0, 1, 2, ...$ which, together with the constraints (1), (2), & (3) given $\xi_{1,-1} = \xi_{2,-1} = 0$ and an initial condition for $\tilde{\omega}_{-1}$, characterize the solution to the optimal policy problem.

Solving this dynamic system with Dynare[9] to get the stationary equilibrium we can look at impulse response functions under the optimal policy. The model is calibrated to quarterly data following Galí (2008)[10], with the exception of the indirect tax process as it does not appear there. This is set as the AR(1) process,

$$\tau_t = c_\tau + \rho_\tau \tau_{t-1} + \epsilon_\tau, \qquad \epsilon_\tau \sim N(0, \sigma_{\epsilon_\tau}^2)$$

Based on the UK data on VAT taxes mentioned above this is calibrated to have an unconditional mean of 0.175, with a first-order autocorrelation of 0.99 (estimated from the quarterly data for

[9] All codes were run in Dynare 4.2.1-2 using Octave 3.2.4

[10] Setting $\epsilon_p = 6$ following Galí (2008), pg 52, gives the wrong numbers when replicating his results, I therefore set it to 6/5 following $http://www.dynare.org/phpBB3/viewtopic.php?f = 1\&t = 2978$

Table 1: Calibrated Micro-Foundation Parameters

	Parameter	Value
Preferences		
Time Discount Rate	β	0.99
Curvature of Consumption	σ	1
Curvature of Labour	φ	1
Production		
Returns to Labour	$1-\alpha$	0.67
Prices and Wages		
Market Power/Markup: Prices	ϵ_p	6/5
Market Power/Markup: Wages	ϵ_w	6/5
Calvo Stickiness: Prices	θ_p	0.66
Calvo Stickiness: Wages	θ_w	0.75
AR(1) process on Productivity Shock		
Autocorrelation	ρ_a	0.9
Standard Deviation	$\sigma_{\epsilon_{as}}$	1
AR(1) process on Indirect Taxes		
Autocorrelation	ρ_τ	0.99
Constant	c_τ	$0.175*(1-\rho_\tau)$
Standard Deviation	σ_{ϵ_τ}	0.025

1991 to 2011; the results are robust to varying this coefficient). The calibrated micro-foundation parameters are shown in Figure 1. All other parameters in the model can be calculated from these micro-foundations.

The impulse response functions to a shock of 0.025 to consumption taxes, which matches the size of each of the changes documented for the UK, are shown in Figure 3. As can been seen optimal monetary policy is to allow the indirect tax increase to pass through as a spike in price inflation. The possibility of second-round effects does not change this policy prescription.

4.3 Optimal Taylor Rules

The optimal policies derived above characterize the behaviour of the economy under optimal monetary policy, however they do not provide any explicit monetary policy rules which we could compare with the actual behaviour of monetary policy. For this we turn to the problem of optimal Taylor rules. The maximization problem to be solved is now the same as in Section 4.2 except that instead of choosing some general i we add the additional constraint that monetary policy be characterized as a Taylor rule on interest rates. Maximization thus involves the choice of the coefficients in the Taylor rule.

The Taylor rule we impose is of a standard form, with the addition of a term allowing monetary policy to react directly to tax changes. Specifically,

$$i_t = c + \rho i_{t-1} + \phi_p \pi_t^p + \phi_w \pi_t^w + \phi_y \tilde{y}_t + \phi_\tau \Delta\tau_t \tag{10}$$

Solving this maximization problem gives us optimal Taylor rules. Using the same calibration as described in the previous section we get that the optimal Talyor rule is

$$i_t = 0.01 + 0.81 i_{t-1} + 1.46\pi_t^p + 0.06\pi_t^w + 0.39\tilde{y}_t - 0.10\Delta\tau_t \tag{11}$$

The optimal Taylor rule for our model is characterized by $\phi_\tau < 0$. Since the calibration of the process on consumption taxes is difficult the results were checked for a variety of parameter values on the autoregressive process, and also for different numbers of lags, with the conclusions on the sign of ϕ_τ being completely robust.

To characterize what the Taylor rule of a central bank that is targeting headline inflation we can think of what would happen had we not assumed the prices immediately reflect indirect tax increases. That is, in an otherwise identical model except where consumer (after-tax) prices are sticky, rather than immeditely reflecting indirect tax increases. This model is developed in Appendix D. In that model optimal monetary policy is to target headline inflation. We find that for a central bank that targets headline inflation the optimal Taylor rule is characterized by $\phi_\tau > 0$ - again this result on the sign of ϕ_τ is robust to various calibrations.

So the sign of ϕ_τ tells us if the central bank is targeting tax-adjusted inflation ($\phi_\tau < 0$) or targeting headline inflation ($\phi_\tau > 0$).

This gives us another way to test what central banks actually do by estimating a Taylor rule from data. For this we turn to the United Kingdom. The UK is chosen as it has changed consumption tax rates four times since 1991; variance in the tax rates being a prerequisite to estimating the ϕ_τ coefficient. Using quarterly data for the period 1991:Q1 to 2011:Q1 we estimate the Taylor rule given by equation (10), but with only one of price and wage inflation at a time (to avoid collinearity problems). Following Clarida, Galí, and Gertler (2000) a forward-looking version, with $E_t\{\pi^p_{t+1}\}$ in place of π^p_t is also estimated. The estimation of the main Taylor rule is done by OLS, while the forward-looking variant uses GMM. In particular the output gap is measured either as the log difference between output and it's (Hodrick-Prescott filtered) trend; or unemployment, based on the theory of Galí (2010). Since interest rates are not set quarterly, both quarterly averages and end of quarter values are used for interest rates. Four measures for inflation are used, the log difference of: Consumer Price Index (CPI) all items, CPI excluding Food and Energy, the GDP deflator, and wage inflation (data from Bank of England, Office for National Statistics, and the Organization for Economic Co-operation and Development; see Appendix A). These inflation measures were calculated both as change from last quarter, and change on year ago. The instruments for expectations of next period price & wage inflation are their own present values and lags, lags of other variables, and present values and lags of M2 money growth & interest rate spreads between 3-month and 5-year or 10-year Treasuries. The results are robust to dropping various of the instruments in the GMM, and to varying the lag lengths used for them (from present value only, to up to three additional lags).

As an example we present the estimation for the basic Taylor rule where the dependent variable is the quarterly average interest rate, inflation is measured as CPI excluding food and fuel, and the output gap is measured by unemployment.

$$\begin{array}{llllll} i_t = & 0.003 & +0.987 i_{t-1} & +0.040 \pi^p_t & -0.080 \tilde{y}_t & +0.234 \Delta\tau_t \\ & (0.002) & (0.030) & (0.084) & (0.045) & (0.127) \end{array}$$

Observe that $\phi_\tau = 0.234$ has a positive sign, and is significant at the 90% level. Using unemployment means we expect the negative sign for the coefficient on the output gap. The insignificance of inflation appears to be due to the small variance of inflation in the UK during this period[11].

The estimated sign on ϕ_τ is always positive, and is statistically significantly different from zero at

[11]See Figure 5 in Appendix A.

a 90% level in the vast majority of cases for the basic Taylor rule[12]. For the forward looking Taylor rule the point estimates for ϕ_τ are largely unchanged, but only significant in a minority of cases. This general loss of significance is unsurprising since we change from OLS to GMM estimation and had just enough observations to begin with. This appears to be a confirmation that central banks target headline inflation and not, as they should, tax-adjusted inflation. However the results should be interpreted with caution since the coefficients on inflation are sometimes insignificant being almost always of small magnitude (in particular in the case of wage inflation and in the forward looking Taylor rule). This appears to be due to a lack of variation in the inflation rate during this period. More complete results of these estimations can be found in Appendix A.

Thus we have further evidence which, while it should be interpreted with caution, suggests that central banks response to indirect tax increases is to target headline inflation. Certainly there is no evidence that that central banks target tax-adjusted inflation. In combination with the evidence of the IMF (2010) and the case of Australia it is clear that central banks target headline inflation, and not, as they should do, target tax-adjusted inflation.

5 Implications for impact on GDP

Changing to tax-adjusted inflation targeting has implications for the effect of an increase in indirect taxes on current GDP and it's evolution over the next few quarters. To see this we consider the impulse response function of output to an increase in indirect taxes of 2.5%.

We compare the impulse response functions of the economy for three cases: under the optimal policy of tax-adjusted inflation targeting, under the optimal Taylor rule for tax-adjusted inflation targeting, and under the optimal Taylor rule related to headline inflation targeting[13].

As seen in Figure 4 the choice of monetary policy has substantial implications for the fall of GDP. Under the optimal monetary policy the increase in indirect taxes of 2.5% leads to a fall in GDP of less than 2%. Under headline inflation targeting the fall in GDP is 4.9%, and takes almost a year (4 periods) to reach what it would have been under optimal monetary policy. Part of this difference however is caused by the inability of a Taylor rule to capture the true optimal policy - under the optimal Taylor rule for tax-adjusted inflation targeting GDP falls by 4.2%. But the fall of GDP under headline inflation targeting is still 1% larger relative to tax-adjusted inflation targeting (comparing the Taylor rules) and lasts for a year. While GDP will fall due to the increase in indirect taxes, the use of headline inflation targeting makes this fall larger[14].

Since current policy is to target headline inflation, the falls in GDP caused by indirect tax increases observed in the literature on fiscal austerity are larger than they should be. Switching to tax-adjusted inflation targeting has implications for GDP large enough (1%) to eliminate the difference in the first year between tax and spending based fiscal consolidation found by Alesina, Favero, and Giavazzi (2012).

[12] The exception being when wage inflation is used

[13] As described in Section 4.3 this third case involves simulating our economy with sticky wages and sticky pre-tax prices under the Taylor rule characterizing headline inflation (the optimal Taylor rule for the economy with sticky consumer prices, where optimal monetary policy is characterized by headline inflation targeting; developed in Appendix D).

[14] Bernanke, Gertler, and Watson (1997) find related empirical results for oil price spikes - while GDP will fall due to an oil price spike, the reaction of monetary policy to oil price spikes causes the fall in GDP to be much larger than otherwise. They do not address the question of whether this reaction represents an optimal trade-off.

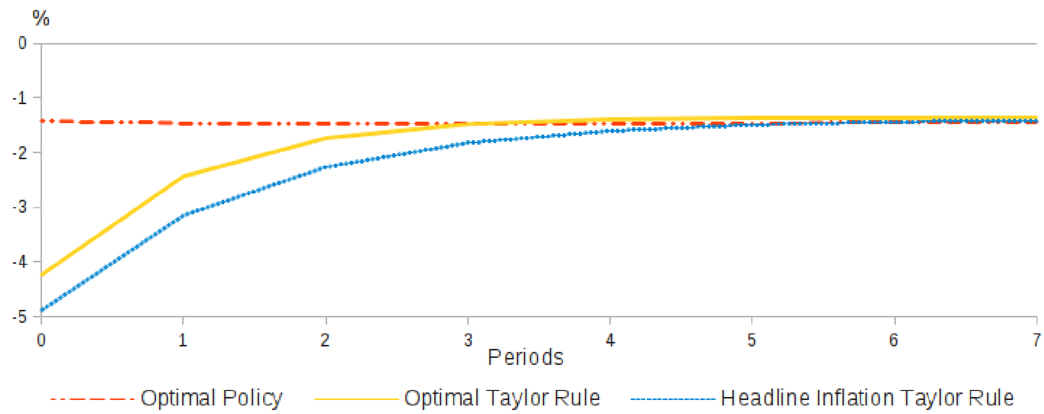


Figure 4: IRFs of output to indirect tax increase of 2.5%
under tax-adjusted vs headline targeting

6 Conclusion

Increases in indirect taxes result in a spike in headline inflation. Central banks fight against this spike – they should allow it. Optimal monetary policy is to allow the spike to occour along with a mild wage inflation. A change from current policies targeting headline inflation to one targeting tax-adjusted inflation would be welfare improving.

The arguments of this paper on how to respond to increases in indirect tax increase have obvious analogues for responding to decreases in indirect taxes. Many of the issues likely extend to other forms of taxation. There are two main limitations to the way taxes have been modeled. Firstly, they are random – so if indirect tax increases are mainly in response to large government budget deficits following a recession, as has been the case recently in many European, certain interactions are missed. Secondly, in the model all changes in indirect taxes are unanticipated.

A switch to targeting tax-adjusted inflation is not just welfare improving. It also has important implications for current fiscal austerity. The fall in GDP associated with indirect tax increases would be less than under current policy. Many countries including the United Kingdom and Spain have increased indirect taxes (VAT & IVA respectively) in the past year by 2.5% or more. The resulting falls of more than 1% of GDP which may be associated with headline inflation targeting are big enough account for almost all of the respective recessions these two countries faced in 2012. In the current climate of fiscal austerity and low growth getting monetary policy right is more important than ever.

7 Acknowledgements

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A Taylor Rule Estimation

This section describes the estimation of the Taylor rule

$$i_t = c + \rho i_{t-1} + \phi \pi_t + \phi_y \tilde{y}_t + \phi_\tau \Delta\tau_t \tag{12}$$

and it's forward-looking version

$$i_t = c + \rho i_{t-1} + \phi^b \pi_{t-1} + \phi^f E_t\{\pi_{t+1}\} + \phi_y \tilde{y}_t + \phi_\tau \Delta\tau_t \tag{13}$$

Estimation of the first equation is done by OLS. Estimation of the second is by GMM, with the instruments for the expectational variables including lags of themselves, lags of all the other variables, and present values and lags of interest rate spreads and M2 growth. The data used is quarterly for the UK from 1991:Q1 to 2011:Q1 (except the M2 growth, which starts in 1996:Q1). The data sources are the Bank of England (BoE: bankofengland.co.uk), the Office for National Statistics (ONS: ons.gov.uk), and the OECD (accessed via FRED: http://research.stlouisfed.org/fred2/). The data series used are

- Interest rate i_t: Follows the Official Bank Rate as set by the Bank of England, since this is not set quarterly both the quarterly average value and the end of quarter value are used alternatively (BoE: IUQABEDR & IUQLBEDR).
- Output gap $\tilde{y}_t$: Either difference between real GDP and it's Hodrick-Prescott filtered trend divided by trend, or the unemployment rate (ONS: ABMI (real GDP); & FRED: GBRURHARMMDSMEI (unemp))
- Inflation π_t: Either price inflation for which one of three measures is used: Consumer Price Index (CPI) all items, CPI ex. Food & Energy, or GDP deflator (FRED: GBRCPIALLQINMEI, GBRCPICORQINMEI, & GBRGDPDEFQISMEI). Or wage inflation: Benchmarked Unit Labor Costs- Total for UK (FRED: GBRULCTOTQPNMEI)
- Other Instruments: Spreads are calculated from 3-month, 5yr and 10yr rates, both the quarterly average and the end of quarter values (BoE: IUQAAJNB, IUQASNPY, IUQAMNPY, IUQAJNB, IUQSNPY, IUQMNPY). M2 growth (BoE: LPQVWYL).

spreads are then given by the differences in the interest rates. Price inflation is log difference between periods of the indexes (wage inflation data is already in % change). The instruments for expectations of next period price & wage inflation are their own present values and lags, lags of other variables, and present values and lags of M2 money growth & interest rate spreads between 3-month and 5-year & 10-year Treasuries. The results are robust to dropping various of the instruments in the GMM, and to varying the lag lengths used for them (from present value only, to up to three additional lags). Estimation is performed with Eviews. Since testing coefficient inequality restrictions (eg. trying to reject H0: $\phi_\tau < 0$) is not yet implemented for VARs in Eviews it is simply checked if the coefficients are statistically significantly different from zero.

Some examples of the regression outputs, chosen as representing some of the most supportative (of the argument that central banks target headline, and not tax-adjusted, inflation) and least supportative results are presented. To interpret the results we note that all variables are measured as percentages. Based on the conventional wisdom we would expect the coefficents to on inflation to always be positive, while those on the output gap would be positive for *ytilde* (deviation of output from trend) and negative for *ytilde*2 (unemployment). As can be seen, those for the output gap behave as might be expected, but the coefficients on inflation suggest that the Bank of England more or less ignores inflation. The later result is likely due to the lack of variation in inflation (by any of the four measures) over this period, as seen in Figure 5.

In the Eview workfile the variables are named as: INTERESTA=quarterly average of interest rate, PIP1A=price inflation calculated from core CPI, PIP2A=price inflation calculated from GDP deflator, PIP3A=price inflation calculated from CPI, PIWA=wage inflation, YTILDE=output gap calculated from GDP with HP-filter, YTILDE2=output gap measured as uninflation.

Table 2: One of the most supportative with basic Taylor Rule

Dependent Variable: INTERESTA
Method: Least Squares
Date: 06/25/12 Time: 17:27
Sample (adjusted): 1992Q1 2011Q1
Included observations: 77 after adjustments
INTERESTA = c + ρ INTERESTA(-1)+ϕ PIP2A
+ϕ_y YTILDE2 + ϕ_τ DELTATAU

	Coefficient	Std. Error	t-Statistic	Prob.
c	0.003890	0.002489	1.562849	0.1225
ρ	0.987470	0.029973	32.94549	0.0000
ϕ	0.040407	0.084314	0.479238	0.6332
ϕ_y	-0.079890	0.044618	-1.790546	0.0776
ϕ_τ	0.233813	0.126527	1.847937	0.0687
R-squared	0.944451	Mean dependent var	0.049949	
Adjusted R-squared	0.941365	S.D. dependent var	0.020924	
S.E. of regression	0.005067	Akaike info criter	-7.669559	
Sum squared resid	0.001848	Schwarz criterion	-7.517363	
Log likelihood	300.2780	Hannan-Quinn crite	-7.608682	
F-statistic	306.0408	Durbin-Watson stat	0.957507	
Prob(F-statistic)	0.000000			

Table 3: One of the least supportative with basic Taylor Rule

Dependent Variable: INTERESTA				
Method: Least Squares				
Date: 06/25/12 Time: 17:26				
Sample (adjusted): 1991Q3 2011Q1				
Included observations: 79 after adjustments				
INTERESTA = c + ρ INTERESTA(-1)+ϕ PIWA				
+ϕ_y YTILDE2 + ϕ_τ DELTATAU				
	Coefficient	Std. Error	t-Statistic	Prob.
c	0.007062	0.002660	2.654870	0.0097
ρ	0.973438	0.026228	37.11393	0.0000
ϕ	-0.248125	0.127341	-1.948508	0.0551
ϕ_y	-0.084531	0.035654	-2.370887	0.0204
ϕ_τ	0.123990	0.102531	1.209301	0.2304
R-squared	0.953124	Mean dependent var		0.051366
Adjusted R-squared	0.950590	S.D. dependent var		0.022471
S.E. of regression	0.004995	Akaike info criter		-7.699620
Sum squared resid	0.001846	Schwarz criterion		-7.549655
Log likelihood	309.1350	Hannan-Quinn crite		-7.639539
F-statistic	376.1576	Durbin-Watson stat		0.952521
Prob(F-statistic)	0.000000			

Table 4: One of the most supportative with forward-looking Taylor Rule

Dependent Variable: INTERESTA
Method: Generalized Method of Moments
Date: 06/08/12 Time: 19:14
Sample (adjusted): 1992Q4 2010Q4
Included observations: 73 after adjustments
Linear estimation with 1 weight update
Estimation weighting matrix: HAC (Bartlett kernel, Newey-West fixed bandwidth = 4.0000)
Standard errors & covariance computed using estimation weighting matri
INTERESTA = c + ρ INTERESTA(-1)+ϕ^b PIP3A(-1)+ϕ^f PIP3A(+1) +ϕ_y YTILDE+ϕ_τ DELTATAU
Instrument specification: INTERESTA(-1) YTILDE DELTATAU PIP3A PIP3A(-1) PIP3A(-2) PIP3A(-3) AVG3M10YSPREAD AVG3M5YSPREAD END3M10YSPREAD END3M5YSPREAD
Constant added to instrument list

	Coefficient	Std. Error	t-Statistic	Prob.
c	0.003320	0.002113	1.571337	0.1208
ρ	0.929470	0.042444	21.89894	0.0000
ϕ^b	0.124897	0.085427	1.462025	0.1484
ϕ^f	-0.154975	0.097269	-1.593256	0.1158
ϕ_y	0.168167	0.081980	2.051320	0.0441
ϕ_τ	0.287413	0.145005	1.982093	0.0516
R-squared	0.923720	Mean dependent var		0.048481
Adjusted R-squared	0.918027	S.D. dependent var		0.018012
S.E. of regression	0.005157	Sum squared resid		0.001782
Durbin-Watson stat	0.846089	J-statistic		11.78057
Instrument rank	12	Prob(J-statistic)		0.067047

Table 5: One of the least supportative with forward-looking Taylor Rule

Dependent Variable: INTERESTA
Method: Generalized Method of Moments
Date: 06/08/12 Time: 19:15
Sample (adjusted): 1992Q4 2010Q4
Included observations: 73 after adjustments
Linear estimation with 1 weight update
Estimation weighting matrix: HAC (Bartlett kernel, Newey-West fixed bandwidth = 4.0000)
Standard errors & covariance computed using estimation weighting matri
INTERESTA = c + ρ INTERESTA(-1)+ϕ PIP1A(-1)+ϕ PIP1A(+1) +ϕ_y YTILDE+ϕ_τ DELTATAU
Instrument specification: INTERESTA(-1) YTILDE DELTATAU PIP1A PIP1A(-1) PIP1A(-2) PIP1A(-3) AVG3M10YSPREAD AVG3M5YSPREAD END3M10YSPREAD END3M5YSPREAD
Constant added to instrument list

	Coefficient	Std. Error	t-Statistic	Prob.
c	0.010051	0.002627	3.826201	0.0003
ρ	0.887277	0.042473	20.89022	0.0000
ϕ^b	-0.177084	0.096864	-1.828181	0.0720
ϕ^f	-0.099413	0.077922	-1.275810	0.2064
ϕ_y	0.208862	0.056631	3.688151	0.0005
ϕ_τ	0.156960	0.109343	1.435481	0.1558
R-squared	0.943323	Mean dependent var		0.048481
Adjusted R-squared	0.939094	S.D. dependent var		0.018012
S.E. of regression	0.004445	Sum squared resid		0.001324
Durbin-Watson stat	0.743380	J-statistic		10.12374
Instrument rank	12	Prob(J-statistic)		0.119537

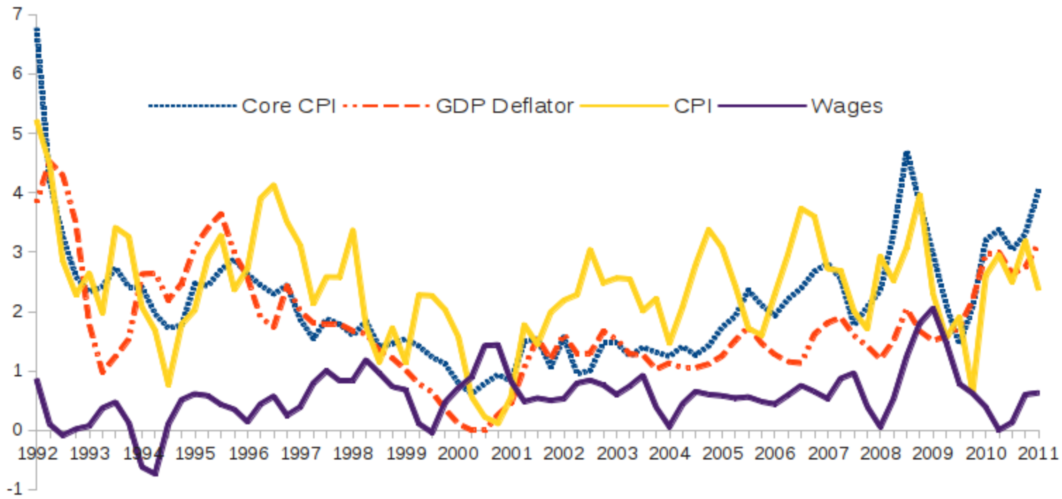

Figure 5: Quarterly inflation at annual rates for the United Kingdom, 1992-2011.

B Sticky Pre-Tax Prices

We modify the basic New Keynesian model introducing a consumption tax. Firms are assumed to set sticky pre-tax prices. Taxes are denoted by $\mathcal{T}$ and assumed to follow a stationary stochastic process (say eg. AR(1)).

B.1 Households

Assume a representative infinitely-lived household, seeking to maximize expected discounted utility by choosing consumption across a continuum of goods indexed by $i \in [0, 1]$ and hours worked, that is to maximize

$$E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, N_t)$$

subject to

$$\int_0^1 (1 + \mathcal{T}_t) P_t(i) C_t(i) di + Q_t B_t \leq B_{t-1} + W_t N_t + T_t \qquad , \forall t$$

$$C_t = \left(\int_0^1 C_t(i)^{\frac{\epsilon - 1}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon - 1}}$$

$$\lim_{T \to \infty} E_t\{B_t\} \geq 0 \qquad , \forall t$$

where N_t is hours worked, C_t is a consumption index, $C_t(i)$ is the quantity of good i consumed, B_t represents purchases of one-period bonds at price Q_t, W_t is nominal wage, T_t is a lump-sum component of income. Using the first-order conditions of maximization, the period budget constraint can be rewritten as

$$(1 + \mathcal{T}_t) P_t C_t + Q_t B_t \leq B_{t-1} + W_t N_t + T_t$$

where P_t is a price index, given by

$$P_t = \left(\int_0^1 P_t(i)^{1-\epsilon} di \right)^{\frac{1}{1-\epsilon}}$$

Using the utility function

$$U(C_t, N_t) = \frac{C_t^{1-\sigma}}{1 - \sigma} - \frac{N_t^{1+\varphi}}{1 + \varphi}$$

the resulting log-linear versions of the optimality conditions are

$$w_t - p_t - \tau_t = \sigma c_t + \varphi n_t \tag{14}$$

$$c_t = E_t\{c_{t+1}\} - \frac{1}{\sigma}(i_t - E_t\{\pi_{t+1}\} - E_t\{\Delta \tau_{t+1}\} - \rho) \tag{15}$$

where $i_t = -log Q_t$ is the short term interest rate, and $rho = -log\beta$ is the (log) discount rate.

B.2 Firms

Assume a continuum of firm indexed by $i \in [0,1]$. Each firm produces a differentiated good, but all using the same technology, given by

$$Y_t(i) = A_t N_t(i)^{1-\alpha} \tag{16}$$

where A_t is technology and is assumed to be exogenous and the same for all firms. Firms face the isoelastic demand schedules given by

$$C_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\epsilon} C_t \tag{17}$$

coming from the households first-order conditions. They take aggregate prices, P_t, and consumption, C_t, as given. Price-stickiness is modelled following Calvo (1983), with each firm able to change it's price only with probability $1-\theta$ in each period. Thus, the problem facing a firm that gets to reoptimize in period t is to choose the price P^* that maximizes the current market value of the profits generated while that price remains effective. That is, that solves

$$\max_{P_t^*} \sum_{k=0}^{\infty} \theta^k E_t\{Q_{t,t+k}(P_t^* Y_{t+k|t} - \Psi_{t+k}(Y_{t+k|t}))\}$$

subject to

$$Y_{t+k|t} = \left(\frac{P_t^*}{P_{t+k}}\right)^{-\epsilon} C_{t+k}$$

The resulting first-order condition of this problem is

$$\sum_{k=0}^{\infty} \theta^k E_t\{Q_{t,t+k} Y_{t+k|t}(P_t^* - \mathcal{M}\Psi'_{t+k}(Y_{t+k|t}))\} = 0$$

where $\mathcal{M} = \frac{\epsilon}{\epsilon-1}$ is the frictionless, or desired, markup (that which would prevail under flexible prices). Dividing through by P_{t-1}, and taking a first-order Taylor expansion of this around the zero inflation steady-state, in logs, yields

$$p_t^* - p_{t-1} = (1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k E_t\{\hat{mc}_{t+k|t} + (p_{t+k} - p_{t-1})\} \tag{18}$$

where $\hat{mc}_{t+k|t} = mc_{t+k|t} - mc$ denotes the log deviation of marginal cost from its steady state value $mc = -\mu$, and where $\mu = -log\mathcal{M}$ is the log of the desired gross markup.

B.3 Aggregate Price Dynamics

Under Calvo-pricing with our definition for the price index, aggregate (after-tax) price dynamics are given by

$$\Pi_t^{1-\epsilon} = \theta \frac{1+\mathcal{T}_t}{1+\mathcal{T}_{t-1}}^{1-\epsilon} + (1-\theta)\frac{1+\mathcal{T}_t}{1+\mathcal{T}_{t-1}}^{1-\epsilon}\left(\frac{P_t^*}{P_{t-1}}\right)^{1-\epsilon}$$

where $\Pi_t = \frac{(1+\mathcal{T}_t)P_t}{(1+\mathcal{T}_{t-1})P_{t-1}}$ is the gross (after-tax) inflation rate.A log-linear approximation to the aggregate price index around the zero inflation steady state yields

$$\pi_t = (\tau_t - \tau_{t-1}) + (1-\theta)(p_t^* - p_{t-1}) \tag{19}$$

B.4 Equilibrium

Market clearing in the goods market requires

$$Y_t(i) = C_t(i) \qquad , \forall i \in [0,1], \ \forall t \tag{20}$$

Letting aggregate output be defined as $Y_t = \left(\int_0^1 Y_t(i)^{\frac{\epsilon-1}{\epsilon}} di\right)^{\frac{\epsilon}{\epsilon-1}}$, it follows that

$$Y_t = C_t \qquad , \forall t \tag{21}$$

Combining this with the consumer's (log) Euler equation gives the equilibrium condition

$$y_t = E_t\{y_{t+1}\} - \frac{1}{\sigma}(i_t - E_t\{\pi_{t+1}\} - E_t\{\Delta\tau_{t+1}\} - \rho) \tag{22}$$

Market clearing in the labour market requires

$$N_t = \int_0^1 N_t(i) di \tag{23}$$

Combining this with firms production function, the demand function, and the goods market clearing condition we get

$$N_t = \left(\frac{Y_t}{A_t}\right)^{\frac{1}{1-\alpha}} \int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{-\frac{\epsilon}{1-\alpha}} di \tag{24}$$

Taking logs,

$$(1-\alpha)n_t = y_t - a_t + d_t \tag{25}$$

where d_t is a measure of price dispersion. It can be shown that d_t, in a neighbourhood of the zero inflation steady state is zero up to a first-order approximation (see, Galí (2008), Appendix 3.3). Thus we get the following relationship

$$y_t = a_t + (1-\alpha)n_t \tag{26}$$

Next an expression is derived for an individual firm's marginal cost in terms of the economy's real marginal cost. The latter is defined by

$$mc_t = (w_t - p_t) - mpn_t \tag{27}$$

$$= (w_t - p_t) - (a_t - \alpha n_t) - log(1-\alpha) \tag{28}$$

$$= (w_t - p_t) - \frac{1}{1-\alpha}(a_t - \alpha y_t) - log(1-\alpha) \tag{29}$$

Using the fact that

$$mc_{t+k|t} = (w_{t+k} - p_{t+k}) - mpn_{t+k|t} \tag{30}$$

$$= (w_{t+k} - p_{t+k}) - \frac{1}{1-\alpha}(a_{t+k} - \alpha y_{t+k|t}) - log(1-\alpha) \tag{31}$$

then

$$mc_{t+k|t} = mc_{t+k} + \frac{\alpha}{1-\alpha}(y_{t+k|t} - y_{t+k}) \tag{32}$$
$$= mc_{t+k} - \frac{\alpha\epsilon}{1-\alpha}(p_t^* - p_{t+k}) \tag{33}$$

where the second equality follows from the demand schedule combined with goods market clearing. Substituting this into (18) and rearranging we have

$$p_t^* - p_{t-1} = (1-\beta\theta)\Theta\sum_{k=0}^{\infty}(\beta\theta)^k E_t\{\widehat{mc}_{t+k}\} + \sum_{k=0}^{\infty}(\beta\theta)^k E_t\{\pi_{t+k} - (\tau_{t+k} - \tau_{t+k-1})\} \tag{34}$$

where $\Theta = \frac{1-\alpha}{1-\alpha+\alpha\epsilon} \leq 1$. Notice that the above discounted sum can be written more compactly as the difference equation

$$p_t^* - p_{t-1} = \beta\theta E_t\{p_{t+1}^* - p_t\} + (1-\beta\theta)\Theta\widehat{mc}_t + \pi_t - (\tau_t - \tau_{t-1}) \tag{35}$$

Which combined with equation defining inflation gives

$$\pi_t = \beta E_t\{\pi_{t+1} - \Delta\tau_{t+1}\} + \lambda\widehat{mc}_t + \Delta\tau_t \tag{36}$$

where $\lambda = \frac{(1-\theta)(1-\beta\theta)}{\theta}\Theta$.

Next, a relation is derived between the economy's real marginal cost and a measure of aggregate economic activity. Notice that, independent of the nature of price setting, average real marginal cost can be expressed as

$$mc_t = (w_t - p_t) - mpn_t \tag{37}$$
$$= (\sigma y_t + \varphi n_t + \tau_t) - (y_t - n_t) - log(1-\alpha) \tag{38}$$
$$= \left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right) y_t - \frac{1+\varphi}{1-\alpha}a_t - log(1-\alpha) + \tau_t \tag{39}$$

where derivation of the second and third equalities make use of the household's optimality condition and the (approximate) aggregate production relation. Furthermore, under flexible prices the real marginal cost is constant and given by $mc = -\mu$. Defining the natural level of output, y_t^n, as the equilibrium level of output under flexible prices

$$mc = \left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right) y_t^n - \frac{1+\varphi}{1-\alpha}a_t - log(1-\alpha) + \tau_t \tag{40}$$

thus implying

$$y_t^n = \psi_{ya}^n a_t + \vartheta_y^n + \frac{1-\alpha}{\sigma(1-\alpha)+\varphi+\alpha}\tau_t \tag{41}$$

where $\vartheta_y^n = -\frac{(1-\alpha)(\mu - log(1-\alpha))}{\sigma(1-\alpha)+\varphi+\alpha} > 0$ and $\psi_{ya}^n = \frac{1+\varphi}{\sigma(1-\alpha)+\varphi+\alpha}$. Subtracting (40) from (39) we obtain

$$\widehat{mc}_t = \left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right)(y_t - y_t^n) \tag{42}$$

Following convention, $\widetilde{y}_t = y_t - y_t^n$ is called the output gap, and measures the distance of output from it's natural (flexible price) counterpart. By combining (42) with (36) we obtain the New Keynesian Phillips Curve,

$$\pi_t = \beta E_t\{\pi_{t+1} - \Delta\tau_{t+1}\} + \kappa\widetilde{y}_t + \Delta\tau_t \tag{43}$$

where $\kappa \equiv \lambda(\sigma + \frac{\varphi+\alpha}{1-\alpha})$. This is one of the key equations describing the equilibrium of the model. The second one, known as the dynamic IS equation, is given by rewriting 22 in terms of the output gap as

$$\widetilde{y}_t = -\frac{1}{\sigma}(i_t - E_t\{\pi_{t+1} - \Delta\tau_{t+1}\} - r_t^n) + E_t\{\widetilde{y}_{t+1}\} \tag{44}$$

where r_t^n is the natural rate of interest, given by

$$\begin{aligned} r_t^n &= \rho + \sigma E_t\{\Delta y_{t+1}^n\} \\ &= \rho + \sigma\psi_{ya}^n E_t\{\Delta a_{t+1}\} + \sigma\psi_{y\tau}^n \Delta\tau_{t+1} \end{aligned}$$

where $\psi_{y\tau}^n = \frac{1-\alpha}{\sigma(1-\alpha)+\varphi+\alpha}$. The addition to equations (43) and (44) of an interest rate rule (an equation for i_t, such as a Taylor rule) completes the model.

C Sticky Pre-Tax Prices and Sticky Wages

We introduce a consumption tax into the sticky prices and sticky wages model. The pre-tax prices are sticky.

C.1 Firms

As in our treatment of the standard sticky prices model, a continuum of firms is assumed, indexes by $i \in [0,1]$, each of which produces a differentiated good with a technology represented by the production function

$$Y_t(i) = A_t N_t(i)^{1-\alpha} \tag{45}$$

where $Y_t(i)$ denotes the output of good i, A_t is an exogenous technology parameter common to all firms, and $N_t(i)$ is an index of labour input used by firm i and defined by

$$N_t(i) \equiv \left[\int_0^1 N_t(i,j)^{1-1/\epsilon_w} dj\right]^{\frac{\epsilon_w}{\epsilon_w - 1}} \tag{46}$$

where $N_t(i,j)$ denotes the quantity of type-j labour employed by firm i in period t. Note that the parameter ϵ_w represents the elasticity substitution among labour varieties. Note also the assumption of a continuum of labour types, indexed by $j \in [0,1]$.

Let $W_t(j)$ denote the wage for type-j labour in period t, for all $j \in [0,1]$. Wages are set by workers. Given wages at time t for the different types of labour services, cost minimization yields a corresponding set of demand schedules for each firm i and labour type j, given the firm's total employment $N_t(i)$

$$N_t(i,j) = \left(\frac{W_t(j)}{W_t}\right)^{-\epsilon_w} N_t(i) \tag{47}$$

for all $i, j \in [0, 1]$, where

$$W_t \equiv \left[\int_0^1 W_t(j)^{1-\epsilon_w} dj\right]^{\frac{1}{1-\epsilon_w}} \tag{48}$$

is an aggregate wage index. Substituting (47) into the definition of $N_t(i)$, one can obtain the convenient aggregation result

$$\int_0^1 W_t(j) N_t(i, j) dj = W_t N_t(i) \tag{49}$$

Hence, and conditional on an optimal allocation of the wage bill among the different types of labour, a firm adjusting it's price in period t will solve the following problem, which is identical to the one analyzed in the standard model with sticky prices

$$\max_{P_t^*} \sum_{k=0}^{\infty} \theta_p^k E_t\{Q_{t,t+k}(P_t^* Y_{t+k|t} - \Phi_{t+k}(Y_{t+k|t}))\} \tag{50}$$

subject to the sequence of demand constraints

$$Y_{t+k|t} = \left(\frac{P_t^*}{P_{t+k}}\right)^{-\epsilon_p} C_{t+k} \tag{51}$$

for $k = 0, 1, 2, \ldots$ where the notation is as before.

As shown previously, the aggregation of the resulting sticky price-setting rules yields, to a first-order approximation and in a neighbourhood of the zero inflation steady state, the following equation for price inflation π_t^p

$$\pi_t^p = \beta E_t\{\pi_{t+1}^p\} + \lambda_p \widehat{mc}_t \tag{52}$$

$$= \beta E_t\{\pi_{t+1}^p\} - \lambda_p \hat{\mu}_t^p \tag{53}$$

where $\hat{\mu}_t^p \equiv \mu_t^p - \mu^p = -\widehat{mc}_t$ and $\lambda_p \equiv \frac{(1-\theta_p)(1-\beta\theta_p)}{\theta_p} \frac{1-\alpha}{1-\alpha+\alpha\epsilon_p}$. Note that, for the sake of symmetry with the wage-inflation equation derived below, the inflation equation is written as a function of the (log) deviation of the average price markup from its desired (or steady state) value, instead of the marginal cost.

C.2 Households

With the introduction of sticky-wages, the households problem now becomes to maximize

$$E_t \left\{\sum_{k=0}^{\infty} (\beta\theta_w)^k U(C_{t+k|t}, N_{t+k|t})\right\} \tag{54}$$

subject to the sequence of labour demand schedules and flow budget constraints that are effective while W_t^* remains in place, ie.

$$N_{t+k|t} = \left(\frac{W_t^*}{W_t + k}\right)^{-\epsilon_w} N_{t+k} \tag{55}$$

$$(1 + \mathcal{T}_{t+k}) P_{t+k} C_{t+k|t} + E_{t+k}\{Q_{t+k,t+k-1} D_{t+k+1|t}\} \leq D_{t+k|t} + W_t^* N_{t+k|t} - T_{t+k} \tag{56}$$

for $k = 0, 1, 2, ...$

The first-order condition associated with the problem above is given by

$$\sum_{k=0}^{\infty}(\beta\theta_w)^k E_t \left\{ N_{t+k|t} U_c(C_{t+k|t}, N_{t+k|t}) \frac{W_t^*}{(1+\mathcal{T}_{t+k})P_{t+k}} + \mathcal{M}_w U_n(C_{t+k|t}, N_{t+k|t}) \right\} = 0 \tag{57}$$

Letting $MRS_{t+k|t}$ be defined as before, the optimality condition above can be rewritten as

$$\sum_{k=0}^{\infty}(\beta\theta_w)^k E_t \left\{ N_{t+k|t} U_c(C_{t+k|t}, N_{t+k|t}) (\frac{W_t^*}{(1+\mathcal{T}_{t+k})P_{t+k}} - \mathcal{M}_w MRS_{t+k|t}) \right\} = 0 \tag{58}$$

Note that in the limiting case of full wage flexibility ($\theta_w = 0$),

$$\frac{W_t^*}{P_t} = \frac{W_t}{P_t} = \mathcal{M}_w MRS_{t|t} \tag{59}$$

for all t. Thus $\mathcal{M}_w$ is the wedge between the real wage and the marginal rate of substitution that prevails in the absence of wage rigidite, ie. the desired gross wage markup.

Note also that in a perfect foresight zero inflation steady state

$$\frac{W^*}{P} = \frac{W}{P} = \mathcal{M}_w MRS \tag{60}$$

Log-linearizing (58) around the steady state yields the following approximate wage setting rule

$$w_t^* = \mu^w + (1-\beta\theta_w)\sum_{k=0}^{\infty}(\beta\theta_w)^k E_t\{mrs_{t+k|t} + p_{t+k} + \tau_{t+k}\} \tag{61}$$

where $\mu^w \equiv log\mathcal{M}_w$.

Using the same utility function as previously, namely

$$U(C,N) = \frac{C^{1-\sigma}}{1-\sigma} - \frac{N^{1+\varphi}}{1+\varphi} \tag{62}$$

the (log) marginal rate of substitution for period $t+k$ for a household that last reset its wage in period t can be written as $mrs_{t+k|t} = \sigma c_{t+k|t} + \varphi n_{t+k|t}$.

Letting $mrs_{t+k} \equiv \sigma c_{t+k} + \varphi n_{t+k}$ define the economy's average marginal rate of substitution,

$$mrs_{t+k|t} = mrs_{t+k} + \varphi(n_{t+k|t} - n_{t+k}) \tag{63}$$

$$= mrs_{t+k} - \epsilon_w\varphi(w_t^* - w_{t+k}) \tag{64}$$

Hence (61) can be rewritten as

$$w_t^* = \frac{1-\beta\theta_w}{1+\epsilon_w\varphi}\sum_{k=0}^{\infty}(\beta\theta_w)^k E_t\{\mu_w + mrs_{t+k} + \epsilon_w\varphi w_{t+k} + p_{t+k} + \tau_{t+k}\} \tag{65}$$

$$= \frac{1-\beta\theta_w}{1+\epsilon_w\varphi}\sum_{k=0}^{\infty}(\beta\theta_w)^k E_t\{(1+\epsilon_w\varphi) - \widehat{\mu}_{t+k}^w \tag{66}$$

$$= \beta\theta_w E_t\{w_{t+t}^*\} + (1-\beta\theta_w)(w_t - (1+\epsilon_w\varphi)^{-1}\widehat{\mu}_{t+k}^w) \tag{67}$$

where $\widehat{\mu}_t^w \equiv \mu_t^w - \mu^w$ denotes the deviations of the economy's (log) average wage markup as $\mu_t^w \equiv (w_t - p_t - \tau_t) - mrs_t$ from its steady state level μ^w.

C.3 Wage Inflation Dynamics

Given the assumed wage setting structure, the evolution of the aggregate wage index is given by

$$W_t = [\theta_w W_{t-1}^{1-\epsilon_w} + (1-\theta_w)(W_t^*)^{1-\epsilon_w}]^{\frac{1}{1-\epsilon_w}} \tag{68}$$

Log-linearizing this around the zero (wage) inflation steady state yields

$$w_t = \theta_w w_{t-1} + (1-\theta_w) w_t^* \tag{69}$$

Combining (67) and (69) and letting $\pi_t^w = w_t - w_{t-1}$ denote wage inflation yields, after some manipulation, the baseline wage inflation equation

$$\pi_t^w = \beta E_t\{\pi_{t+1}^w\} - \lambda_w \widehat{\mu}_t^w \tag{70}$$

where $\lambda_w \equiv \frac{(1-\theta_w)(1-\beta\theta_w)}{\theta_w(1+\epsilon_w\varphi)}$. Note that this wage inflation equation has a form analogous to that describing the dynamics of price inflation.

In this model the wage inflation equation (70) replaces condition $w_t - p_t = mrs_t$, one of the optimality conditions associated with the household's problem in the model with just sticky-prices. The imperfect adjustment of nominal wages will generally drive a wedge between the real wage and the marginal rate of substitution of each household and, as a result, between the average real wage and the average marginal rate of substitution, leading to variations in the average wage markup.

C.4 Other Optimality Conditions

In addition to the optimal wage setting condition (58), the solution to the above household's problem also yields a conventional Euler equation, which when log-linearized takes the same form as in the sticky prices model, namely

$$c_t = E_t\{c_{t+1}\} - \frac{1}{\sigma}(i_t - E_t\{\pi_{t+1}^p\} - \rho) \tag{71}$$

C.5 Equilibrium

The output gap is once more defined as $\tilde{y}_t \equiv y_t - y_t^n$, although the natural level of output, y_t^n, is now that which would occour in the absence of both price and wage stickiness. The *real wage gap*, is again defined as $\tilde{\omega}_t \equiv \omega_t - \omega_t^n$, where however now $\omega_t \equiv w_t - p_t - \tau_t$, denotes the real wage, and where ω_t^n is the natural real wage, the real wage that would prevail in the absence of nominal rigidities, and which is given by

$$\begin{aligned}\omega_t^n &= log(1-\alpha) + (y_t^n - n_t^n) - \mu^p \\ &= log(1-\alpha) + \psi_{\omega a}^n a_t - \psi_{\omega\tau}^n \tau_t - \mu^p\end{aligned}$$

where $\psi_{\omega a}^n \equiv \frac{1-\alpha\psi_{ya}^n}{1-\alpha} \geq 0$ and $\psi_{\omega\tau}^n \equiv \frac{1-\alpha\psi_{y\tau}^n}{1-\alpha} \geq 0$.

First, relate the average price markup to the output and real wage gaps. Using the fact that $\mu_t^p = mpn_t - \omega_t$,

$$\begin{aligned}
\widehat{\mu}_t^p &= (mpn_t - \omega_t) - \mu^p && (72)\\
&= (\tilde{y}_t - \tilde{n}_t) - \widehat{\omega}_t && (73)\\
&= -\frac{\alpha}{1-\alpha}\tilde{y}_t - \tilde{\omega}_t && (74)
\end{aligned}$$

As with the model with consumption taxes and sticky pre-tax prices (but flexible wages) we have

$$\begin{aligned}
\pi_t^p &= \beta E_t\{\pi_{t+1}^p - \Delta\tau_{t+1}\} + \lambda_p \widehat{mc}_t + \Delta\tau_t && (75)\\
&= \beta E_t\{\pi_{t+1}^p - \Delta\tau_{t+1}\} - \lambda_p \hat{\mu}_t^p + \lambda_p \hat{\tau}_t + \Delta\tau_t && (76)
\end{aligned}$$

as now $mc_t = -\mu_t^p + \tau_t$ (since economy's avg marginal cost is $mc_t = (w_t - p_t) - mpn_t$ and consumer optimization implies $w_t - p_t - \tau_t = -\mu_t^p + mpn_t$).

Hence, combining (76) & (74) yields the following equation for price inflation as a function of the output and real wage gaps

$$\pi_t^p = \beta E_t\{\pi_{t+1}^p - \Delta\tau_{t+1}\} + \kappa_p \tilde{y}_t + \lambda_p \tilde{\omega}_t + \lambda_p \hat{\tau}_t + \Delta\tau_t \quad (77)$$

where $\kappa_p \equiv \frac{\alpha\lambda_p}{1-\alpha}$.

Similarly,

$$\begin{aligned}
\hat{\mu}_t^w &= \omega_t - mrs_t - \mu^w && (78)\\
&= \tilde{\omega}_t - (\sigma\tilde{y}_t + \varphi\tilde{n}_t && (79)\\
&= \tilde{\omega}_t - \left(\sigma + \frac{\varphi}{1-\alpha}\tilde{y}_t\right) && (80)
\end{aligned}$$

Combining, (70) and (80) yields an analogous version of the wage inflation equation in terms of the output and real wage gaps

$$\pi_t^w = \beta E_t\{\pi_{t+1}^w\} + \kappa_w \tilde{y}_t - \lambda_w \tilde{\omega}_t \quad (81)$$

where $\kappa_w \equiv \lambda_w(\sigma + \frac{\varphi}{1-\alpha})$.

In addition, there is an identity relating the changes in the wage gap to price inflation , wage inflation, and the natural wage

$$\tilde{\omega}_t \equiv \tilde{\omega}_{t-1} + \pi_t^w - \pi_t^p - \Delta\omega_t^p \quad (82)$$

which is unchanged, since $\pi_t^p = (p_t + \tau_t) - (p_{t-1} + \tau_{t-1})$ is after-tax inflation.

In order to complete the non-policy block of the model, equilibrium conditions (77), (81), and (82) must be supplemented with a dynamic IS equation, like that of the sticky prices only model which can be derived by combining the goods market clearing condition $y_t = c_t$ with Euler equation (71). The resulting equation is rewritten in terms of the output gap as

$$\tilde{y}_t = E_t\{\tilde{y}_{t+1}\} - \frac{1}{\sigma}(i_t - E_t\{\pi_{t+1}^p\} - r_t^n) \quad (83)$$

where the natural interest rate $r_t^n \equiv \rho + \sigma E_t\{\Delta y_{t+1}^n\}$ should now be understood as the rate prevailing in an equilibrium with flexible wages and prices.

D Sticky Wages and Sticky After-tax prices

We take the standard New Keynesian model and add consumption taxes with firms setting sticky after-tax prices. This model is intended to capture the mindset of central banks in targeting headline inflation. Treatment is based on Galí (2008) Chapter 6, which in turn introduces sticky wages following Erceg, Henderson, and Levin (2000). We describe the micro-foundations of the model, and then derive the system of equations derived from these which describe the dynamic behaviour of the system. The sufficient conditions for the optimal monetary policy are then given and their implication of inflation targeting is derived. Taxes are denoted by $\mathcal{T}$ and assumed to follow a stationary stochastic process (say eg. AR(1)). It is assumed that tax revenue is simply returned as a lump-sum transfer. Lower-case letters are used throughout to denote the log-deviations from steady-state of the corresponding upper-case letter. We begin by looking at the firms problem.

D.1 Firms

There is a continuum of consumption goods indexed by $i \in [0,1]$, each of which is produced by a different firm, all of which have access to the same production funtion, given by $Y_t(i) = A_t N_t(i)^{1-\alpha}$ where $Y_t(i)$ denotes the output of good i, A_t is an exogenous technology parameter common to all firms, and $N_t(i)$ is an index of labour input used by firm i and defined by

$$N_t(i) \equiv \left[\int_0^1 N_t(i,j)^{1-1/\epsilon_w} dj\right]^{\frac{\epsilon_w}{\epsilon_w - 1}}$$

where $N_t(i,j)$ denotes the quantity of type-j labour employed by firm i in period t. Note that the parameter ϵ_w represents the elasticity substitution among labour varieties. Note also the assumption of a continuum of labour types, indexed by $j \in [0,1]$.

Let $W_t(j)$ denote the wage for type-j labour in period t, for all $j \in [0,1]$. Wages are set by workers. Given wages at time t for the different types of labour services, cost minimization yields a corresponding set of demand schedules for each firm i and labour type j, given the firm's total employment $N_t(i)$

$$N_t(i,j) = \left(\frac{W_t(j)}{W_t}\right)^{-\epsilon_w} N_t(i)$$

for all $i, j \in [0,1]$, where $W_t \equiv \left[\int_0^1 W_t(j)^{1-\epsilon_w} dj\right]^{\frac{1}{1-\epsilon_w}}$ is an aggregate wage index.

Each period firms are allowed to change prices only with probability $1-\theta$. Hence, and conditional on an optimal allocation of the wage bill among the different types of labour, a firm adjusting it's price in period t maximizes its expected profits during the time in which this price, P_t^*, is expected to be in place, Thus it faces the following problem, which is identical to the one analyzed in the standard model with sticky prices

$$\max_{P_t^*} \sum_{k=0}^{\infty} \theta_p^k E_t\{Q_{t,t+k}(P_t^* Y_{t+k|t} - \Phi_{t+k}(Y_{t+k|t}))\}$$

subject to the sequence of demand constraints

$$Y_{t+k|t} = \left(\frac{P_t^*}{P_{t+k}}\right)^{-\epsilon_p} C_{t+k}$$

for $k = 0, 1, 2, ...$ where the notation is as before.

The resulting FOC of this problem is:

$$\sum_{k=0}^{\infty} \theta^k E_t\{Q_{t,t+k} Y_{t+k|t}(\frac{P_t^*}{1+\mathcal{T}_t} - \mathcal{M}\Psi'_{t+k}(Y_{t+k|t}))\} = 0$$

where $\mathcal{M} = \frac{\epsilon}{\epsilon-1}$ is the frictionless, or desired, markup (that which would prevail under flexible prices). Dividing through by $P_{t-1}/(1+\mathcal{T}_{t-1})$, and taking a first-order Taylor expansion of this around the zero inflation steady-state, in logs, yields

$$p_t^* - p_{t-1} = (\tau_t - \tau_{t-1}) + (1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k E_t\{\hat{mc}_{t+k|t} + (p_{t+k} - p_{t-1}) - (\tau_{t+k} - \tau_{t-1})\} \quad (84)$$

where $\hat{mc}_{t+k|t} = mc_{t+k|t} - mc$ denotes the log deviation of marginal cost from its steady state value $mc = -\mu$ (note that here it does not include taxes), and where $\mu = -log\mathcal{M}$ is the log of the desired gross markup. τ_t denotes the log deviation from steady state of $(1+\mathcal{T}_t$.

D.2 Households

To introduce sticky-wages we have assumed that each household supplies a differentiated labour type indexed by $j \in [0,1]$. These are then later aggregated into a single labour input used in production via a Dixit-Stiglitz aggregator. Every period with probability $1-\theta_w$ the household gets to choose a wage, otherwise it is stuck with the wage it had last period. Households maximizes their expected discounted utility choosing hours worked, consumption, and savings. Consumption is given by a constant elasticity of substitution index of consumption across a continuum of goods indexed by $i \in [0,1]$, that is $C_t = \left(\int_0^1 C_t(i)^{\frac{\epsilon-1}{\epsilon}} di\right)^{\frac{\epsilon}{\epsilon-1}}$, where $C_t(i)$ is consumption of differentiated good i. The problem of a household that gets to set it's wage in period t thus becomes to maximize

$$E_t\left\{\sum_{k=0}^{\infty}(\beta\theta_w)^k U(C_{t+k|t}, N_{t+k|t})\right\} \quad (85)$$

subject to the sequence of labour demand schedules and flow budget constraints that are effective while W_t^* remains in place, ie.

$$N_{t+k|t} = \left(\frac{W_t^*}{W_t + k}\right)^{-\epsilon_w} N_{t+k} \quad (86)$$

$$P_{t+k}C_{t+k|t} + E_{t+k}\{Q_{t+k,t+k-1}D_{t+k+1|t}\} \leq D_{t+k|t} + W_t^* N_{t+k|t} - T_{t+k} \quad (87)$$

for $k = 0, 1, 2, ...$

The first-order condition associated with the problem above is given by

$$\sum_{k=0}^{\infty}(\beta\theta_w)^k E_t\left\{N_{t+k|t}U_c(C_{t+k|t}, N_{t+k|t})\frac{W_t^*}{P_{t+k}} + \mathcal{M}_w U_n(C_{t+k|t}, N_{t+k|t})\right\} = 0 \quad (88)$$

where $\mathcal{M} \equiv \frac{\epsilon_w}{\epsilon_w - 1}$.

Letting $MRS_{t+k|t} \equiv -\frac{U_n(C_{t+k|t}, N_{t+k|t})}{U_c(C_{t+k|t}, N_{t+k|t})}$ denote the marginal rate of substitution between consumption and hours in period $t+k$ for the household resetting the wage in period t, the optimality condition above can be rewritten as

$$\sum_{k=0}^{\infty} (\beta\theta_w)^k E_t \left\{ N_{t+k|t} U_c(C_{t+k|t}, N_{t+k|t}) (\frac{W_t^*}{P_{t+k}} - \mathcal{M}_w MRS_{t+k|t}) \right\} = 0 \tag{89}$$

Note that in the limiting case of full wage flexibility ($\theta_w = 0$),

$$\frac{W_t^*}{P_t} = \frac{W_t}{P_t} = \mathcal{M}_w MRS_{t|t} \tag{90}$$

for all t. Thus $\mathcal{M}_w$ is the wedge between the real wage and the marginal rate of substitution that prevails in the absence of wage rigidities, ie. the desired gross wage markup.

Note also that in a perfect foresight zero inflation steady state

$$\frac{W^*}{P} = \frac{W}{P} = \mathcal{M}_w MRS \tag{91}$$

Log-linearizing (89) around that steady state yields, after some algebraic manipulation, the following approximate wage setting rule

$$w_t^* = \mu^w + (1 - \beta\theta_w) \sum_{k=0}^{\infty} (\beta\theta_w)^k E_t\{mrs_{t+k|t} + p_{t+k}\} \tag{92}$$

where $\mu^w \equiv log\mathcal{M}_w$.

Using the same utility function as previously, namely

$$U(C, N) = \frac{C^{1-\sigma}}{1-\sigma} - \frac{N^{1+\varphi}}{1+\varphi} \tag{93}$$

the (log) marginal rate of substitution for period $t+k$ for a household that last reset its wage in period t can be written as $mrs_{t+k|t} = \sigma c_{t+k|t} + \varphi n_{t+k|t}$.

Letting $mrs_{t+k} \equiv \sigma c_{t+k} + \varphi n_{t+k}$ define the economy's average marginal rate of substitution,

$$\begin{aligned} mrs_{t+k|t} &= mrs_{t+k} + \varphi(n_{t+k|t} - n_{t+k}) && (94) \\ &= mrs_{t+k} - \epsilon_w \varphi (w_t^* - w_{t+k}) && (95) \end{aligned}$$

Hence (92) can be rewritten as

$$\begin{aligned} w_t^* &= \frac{1-\beta\theta_w}{1+\epsilon_w\varphi} \sum_{k=0}^{\infty} (\beta\theta_w)^k E_t\{\mu_w + mrs_{t+k} + \epsilon_w \varphi w_{t+k} + p_{t+k}\} && (96) \\ &= \frac{1-\beta\theta_w}{1+\epsilon_w\varphi} \sum_{k=0}^{\infty} (\beta\theta_w)^k E_t\{(1+\epsilon_w\varphi) - \widehat{\mu}_{t+k}^w && (97) \\ &= \beta\theta_w E_t\{w_{t+t}^*\} + (1-\beta\theta_w)(w_t - (1+\epsilon_w\varphi)^{-1} \widehat{\mu}_{t+k}^w) && (98) \end{aligned}$$

where $\widehat{\mu}_t^w \equiv \mu_t^w - \mu^w$ denotes the deviations of the economy's (log) average wage markup as $\mu_t^w \equiv (w_t - p_t) - mrs_t$ from its steady state level μ^w.

D.3 Price and Wage Inflation Dynamics

Given the assumed wage setting structure, the evolution of the aggregate wage index is given by

$$W_t = [\theta_w W_{t-1}^{1-\epsilon_w} + (1-\theta_w)(W_t^*)^{1-\epsilon_w}]^{\frac{1}{1-\epsilon_w}} \tag{99}$$

Log-linearizing this around the zero (wage) inflation steady state yields

$$w_t = \theta_w w_{t-1} + (1-\theta_w)w_t^* \tag{100}$$

Combining (98) and (100) and letting $\pi_t^w = w_t - w_{t-1}$ denote wage inflation yields, after some manipulation, the baseline wage inflation equation

$$\pi_t^w = \beta E_t\{\pi_{t+1}^w\} - \lambda_w \widehat{\mu}_t^w \tag{101}$$

where $\lambda_w \equiv \frac{(1-\theta_w)(1-\beta\theta_w)}{\theta_w(1+\epsilon_w\varphi)}$. Note that this wage inflation equation has a form analogous to that describing the dynamics of price inflation.

An equation for the evolution of the aggregate price level (an index of the prices for the individual goods) under Calvo pricing is given by

$$\Pi_t^{1-\epsilon} = \theta + (1-\theta)\left(\frac{P_t^*}{P_{t-1}}\right)^{1-\epsilon}$$

where $\Pi_t = \frac{P_t}{P_{t-1}}$ is the gross inflation rate.

D.4 Equilibrium

Market clearing in the model involves market clearing for each of the consumption goods, $C_t(i) = Y_t(i)$, $\forall\ i \in [0,1]$, $\forall\ t$, and in the labour market $N_t = \int_0^1 N_t(i)di$. The output gap is once more defined as $\tilde{y}_t \equiv y_t - y_t^n$, although the natural level of output, y_t^n, is now that which would occour in the absence of both price and wage stickiness. A new variable, the *real wage gap*, is defined as $\tilde{\omega}_t \equiv \omega_t - \omega_t^n$, where $\omega_t \equiv w_t - p_t$, denotes the real wage, and where ω_t^n is the natural real wage, the real wage that would prevail in the absence of nominal rigidities, and which is given by

$$\omega_t^n = log(1-\alpha) + (y_t^n - n_t^n) - \mu^p = log(1-\alpha) + \psi_{\omega a}^n a_t - \mu^p$$

where $\psi_{\omega a}^n \equiv \frac{1-\alpha\psi_{ya}^n}{1-\alpha} \geq 0$ and $\psi_{ya}^n \equiv \frac{1+\varphi}{\sigma(1-\alpha)+\varphi+\alpha}$.

D.5 Derivation of System of Equations

The consumer's (log) Euler equation is given by the equilibrium condition

$$y_t = E_t\{y_{t+1}\} - \frac{1}{\sigma}(i_t - E_t\{\pi_{t+1} - \rho) \tag{102}$$

As before we have the following relationship

$$y_t = a_t + (1-\alpha)n_t \tag{103}$$

Comining this with goods market clearance we have

$$c_t = E_t\{c_{t+1}\} - \frac{1}{\sigma}(i_t - E_t\{\pi^p_{t+1}\} - \rho) \tag{104}$$

The output gap is once more defined as $\tilde{y}_t \equiv y_t - y^n_t$, although the natural level of output, y^n_t, is now that which would occour in the absence of both price and wage stickiness. The *real wage gap*, is again defined as $\tilde{\omega}_t \equiv \omega_t - \omega^n_t$, where $\omega_t \equiv w_t - p_t$, denotes the real wage, and where ω^n_t is the natural real wage, the real wage that would prevail in the absence of nominal rigidities, and which is given by

$$\begin{aligned}\omega^n_t &= log(1-\alpha) + (y^n_t - n^n_t) - (\mu^p + \tau_t) \\ &= log(1-\alpha) + \psi^n_{\omega a} a_t - \psi^n_{\omega\tau}\tau_t - (\mu^p + \tau_t)\end{aligned}$$

where $\psi^n_{\omega a} \equiv \frac{1-\alpha\psi^n_{ya}}{1-\alpha} \geq 0$ and $\psi^n_{\omega\tau} \equiv \frac{1-\alpha\psi^n_{y\tau}}{1-\alpha} \geq 0$.

First, relate the average price markup to the output and real wage gaps. Using the fact that $\mu^p_t = mpn_t - \omega_t$,

$$\widehat{\mu}^p_t = (mpn_t - \omega_t) - \mu^p \tag{105}$$

$$= (\tilde{y}_t - \tilde{n}_t) - \widehat{\omega}_t \tag{106}$$

$$= -\frac{\alpha}{1-\alpha}\tilde{y}_t - \tilde{\omega}_t \tag{107}$$

As with the model with consumption taxes and sticky after-tax prices (but flexible wages) we have

$$\pi^p_t = \beta E_t\{\pi^p_{t+1} - \Delta\tau_{t+1}\} + \lambda_p \widehat{mc}_t \tag{108}$$

$$= \beta E_t\{\pi^p_{t+1} - \Delta\tau_{t+1}\} - \lambda_p \hat{\mu}^p_t + \lambda_p \hat{\tau}_t \tag{109}$$

as now $mc_t = -\mu^p_t - \tau_t$ (since economy's avg marginal cost is $mc_t = (w_t - p_t) - mpn_t$ and consumer optimization implies $w_t - p_t = -\mu^p_t + mpn_t$).

Hence, combining (109) & (107) yields the following equation for price inflation as a function of the output and real wage gaps

$$\pi^p_t = \beta E_t\{\pi^p_{t+1}\} + \kappa_p \tilde{y}_t + \lambda_p \tilde{\omega}_t + \lambda_p \hat{\tau}_t \tag{110}$$

where $\kappa_p \equiv \frac{\alpha\lambda_p}{1-\alpha}$.

Similarly,

$$\hat{\mu}^w_t = \omega_t - mrs_t - \mu^w \tag{111}$$

$$= \tilde{\omega}_t - (\sigma\tilde{y}_t + \varphi\tilde{n}_t \tag{112}$$

$$= \tilde{\omega}_t - \left(\sigma + \frac{\varphi}{1-\alpha}\tilde{y}_t\right) \tag{113}$$

Combining, (101) and (113) yields an analogous version of the wage inflation equation in terms of the output and real wage gaps

$$\pi^w_t = \beta E_t\{\pi^w_{t+1}\} + \kappa_w \tilde{y}_t - \lambda_w \tilde{\omega}_t \tag{114}$$

where $\kappa_w \equiv \lambda_w(\sigma + \frac{\varphi}{1-\alpha})$.

In addition, there is an identity relating the changes in the wage gap to price inflation , wage inflation, and the natural wage

$$\tilde{\omega}_t \equiv \tilde{\omega}_{t-1} + \pi_t^w - \pi_t^p - \Delta\omega_t^p \tag{115}$$

In order to complete the non-policy block of the model, equilibrium conditions (110), (114), and (115) must be supplemented with a dynamic IS equation, like that of the sticky prices only model which can be derived by combining the goods market clearing condition $y_t = c_t$ with Euler equation (104). The resulting equation is rewritten in terms of the output gap as

$$\tilde{y}_t = E_t\{\tilde{y}_{t+1}\} - \frac{1}{\sigma}(i_t - E_t\{\pi_{t+1}^p\} - r_t^n) \tag{116}$$

where the natural interest rate,

$$\begin{aligned} r_t^n & \equiv \rho + \sigma E_t\{\Delta y_{t+1}^n\} \\ &= \rho + \sigma\varphi_{ya}^n E_t\{\Delta a_{t+1}\} - \tfrac{1-\alpha}{\sigma(1-\alpha)+\varphi+\alpha} E_t\{\Delta\tau_{t+1}\} \end{aligned}$$

should now be understood as the rate prevailing in an equilibrium with flexible wages and prices.

D.6 System of Equations

Summarizing, from the micro-foundations, we derived the following system of equations characterizing the dynamic behaviour of the model. The first equation is the NKPC

$$\pi_t^p = \beta E_t\{\pi_{t+1}^p\} + \kappa_p \tilde{y}_t + \lambda_p \tilde{\omega}_t - \lambda_p \hat{\tau}_t \tag{117}$$

Notice that $\omega_t \equiv w_t - p_t$, and p_t reacts to τ_t but w_t doesn't, hence ω_t does; this is why NKPC for prices now has the $\lambda_p(om\tilde{e}ga_t - \hat{\tau}_t$ term, which with flexible wages would be zero. Next, the NKPC for wages

$$\pi_t^w = \beta E_t\{\pi_{t+1}^w\} + \kappa_w \tilde{y}_t - \lambda_w \tilde{\omega}_t \tag{118}$$

In addition, there is an identity relating the changes in the wage gap to price inflation , wage inflation, and the natural wage

$$\tilde{\omega}_t = \tilde{\omega}_{t-1} + \pi_t^w - \pi_t^p - \Delta\omega_t^n \tag{119}$$

we once again get the dynamic IS equation

$$\tilde{y}_t = E_t\{\tilde{y}_{t+1}\} - \frac{1}{\sigma}(i_t - E_t\{\pi_{t+1}^p\} - r_t^n) \tag{120}$$

where as in case without sticky wages

$$r_t^n = \rho - \sigma E_t\{\Delta y_{t+1}^n\} = \rho - \sigma\psi_{ya}^n E_t\{\Delta a_{t+1}\} + \sigma\psi_{y\tau}^n E_t\{\Delta\tau_{t+1}\}$$

however this should now be understood as the rate prevailing in an equilibrium with both flexible wages and prices. Where τ_t denotes the log deviation from steady state of $1 + \mathcal{T}_t$, $\widetilde{y}_t$ is the output gap (the difference between actual output y_t and the natural level y_t^n which would result under flexible prices), r_t^n is the natural interest rate (that associated with the flexible price output y_t^n), and i_t is the nominal interest rate. Together with a monetary policy rule defining the evolution of i_t these equations form a system of equations that fully describe the evolution of the model.

D.7 Optimal Taylor Rules

When considering optimal monetary policy one further assumption is required. Following the literature, it is assumed that the distortion caused by the market power of the firms and labour arising from monopolistic competition is not something to be considered by monetary authorities. For this reason a wage-subsidy is assumed that makes the equilibrium under flexible prices efficient. With this wage-subsidy in place the decentralized equilibrium is efficient, corresponding to that which would be chosen by as social planner (see Appendix Galí (2008)). For our purposes, the wage-subsidy is further assumed to balance the distortions of the consumption tax to avoid monetary policy trying to fight this. Monetary policy aims to avoid distortions arising from sticky-prices, both from the average marginal costs diverging from their optimal level, and from distortions in relative prices. Thus, optimal policy will be that which keeps the output gap closed, $\widetilde{y}_t$, for all t. Observe that the natural level of output (from which the output gap is measured) is the same as in the sticky pre-tax prices case.

It can be shown (see Galí (2008) Appendix 6.2; the proof carries over directly to the case with consumption taxes and sticky after-tax prices) that, based on an approximation of the utility function, the welfare expressed as a fraction of steady state consumption are given by

$$\mathbb{W} = \frac{1}{2}E_0\sum_{t=0}^{\infty}\beta^t\left((\sigma + \frac{\varphi+\alpha}{1-\alpha})\tilde{y}_t^2 + \frac{\epsilon_p}{\lambda_p}(\pi_t^p)^2 + \frac{\epsilon_w(1-\alpha)}{\lambda_w}(\pi_t^w)^2\right) + t.i.p \tag{121}$$

where $t.i.p.$ collects various terms that are independent of policy. Ignoring the latter terms we can express the average period welfare loss as

$$\mathbb{L} = (\sigma + \frac{\varphi+\alpha}{1-\alpha})var(\tilde{y}_t) + \frac{\epsilon_p}{\lambda_p}var(\pi_t^p) + \frac{\epsilon_w(1-\alpha)}{\lambda_w}var(\pi_t^w) \tag{122}$$

Taking a primal approach to characterizing optimal monetary policy, that is, characterizing the behaviour of the economy under the optimal policy without actually calculating what form it takes as an interest rate rule. Optimal monetary policy is given by the central bank seeking to maximize (121) subject to (117), (118) & (119) for $t = 0, 1, 2, ...$ Let $\{\xi_{1,t}\}$, $\{\xi_{2,t}\}$, & $\{\xi_{3,t}\}$ denote the sequence of Lagrange multipliers associated with the previous constraints, respectively. The optimality conditions for the optimal policy are thus given by

$$\left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right)\tilde{y}_t + \kappa_p\xi_{1,t} + \kappa_w\xi_{2,t} = 0 \tag{123}$$

$$\frac{\epsilon_p}{\lambda_p}\pi_t^p - \Delta\xi_{1,t} + \xi_{3,t} = 0 \tag{124}$$

$$\frac{\epsilon_w(1-\alpha)}{\lambda_w}\pi_t^w - \Delta\xi_{2,t} - \xi_{3,t} = 0 \tag{125}$$

$$\lambda_p\xi_{1,t} - \lambda_w\xi_{2,t} + \xi_{3,t} - \beta E_t\{\xi_{3,t+1}\} = 0 \tag{126}$$

for $t = 0, 1, 2, ...$ which, together with the constraints (117), (118), & (119) given $\xi_{1,-1} = \xi_{2,-1} = 0$ and an initial condition for $\tilde{\omega}_{-1}$, characterize the solution to the optimal policy problem.

Adding the further restriction that monetary policy takes the form of a Taylor Rule, specifically one of the form

$$i_t = c + \rho i_{t-1} + \phi_p\pi_t^p + \phi_w\pi_t^w + \phi_y\tilde{y}_t + \phi_\tau\Delta\tau_t \tag{127}$$

the maximization problem to be solved is now to maximize (121) subject to (117), (118), (119), & (127). Maximization thus involves the choice of the coefficients in the Taylor rule, and is done using Dynare using the same calibration as that used in the rest of the paper.