Syllabus "Mathematics II" **2013/2014**

Code: 102345
ECTS: 6

Degree	Year	Semester
950 Business Management and Administration (EHEA Degree)	1	2

Contact

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Language

English

Prerequisites

To follow properly this course, it is necessary a correct handle of the basic mathematical concepts and tools, as well as of the fundamental notions of continuity, differentiability, and graphical representation of real functions of one real variable studied in the course of Mathematics I.

Objectives

This course introduces students to the study of linear algebra and functions of several variables, with emphasis on their applications in economics. Students should not only acquire and assimilate new mathematical knowledge, but also be able to apply them in quantitative analysis in economics and business.
Therefore, the purpose of the course is that students become familiar with basic mathematical concepts to be used in the study of economic theory and analysis.

Specifically the objectives are intended to achieve are:

1. To familiarize students with the n-dimensional Euclidean space.
2. Working with determinants and matrices.
3. Solving systems of linear equations.
4. Understanding the functions of several variables and their role in more complex economic models.
5. Geometric representation of functions of two variables using contour maps.
6. Understand the concepts of limit of a function at a point and of a continuous function.
7. Understanding the Weierstrass theorem.
8. To familiarize students with the partial derivatives of functions of several variables and the concept of differentiability.
9. Using partial derivatives to obtain the slope of the contour at one point and to perform comparative statics exercises.
10. Solving optimization problems without constraints and with equality constraints.

Competences and learning outcomes

Code: 1991:E01
Type: E
Competence: Demonstrate understanding of the language and some methods of mathematical proof.
Outcome of learning:

1991:E01.01 - Discuss and solve systems of linear equations.
1991:E01.02 - Calculate decompositions of matrices and determinants.
1991:E01.03 - Work with different bases of finite dimensional vector spaces.
1991:E01.04 - Classify matrices and linear applications according to various criteria (range, diagonal and Jordan forms).
1991:E01.05 - Know the basic results of differential calculus in several real variables.
1991:E01.06 - Calculate derivatives of functions using the chain rule, implicit function theorem, etc..
1991:E01.07 - Compute and study the extremes of functions.

Code: 1991:E39
Type: E
Competence: Apply mathematical tools to simplify complex business and economic situations.
Outcome of learning:

1991:E39.01 - Apply the inverse function theorem and the implicit function theorem to specific problems.
1991:E39.02 - Set and solve analytically optimization economic problems.

Code: 1991:T01
Type: T
Competence: Ability to communicate orally and in writing in Catalan, Spanish and English.
Outcome of learning:
1991:T01.00 - Ability to communicate orally and in writing in Catalan, Spanish and English.

Code: 1991:T04
Type: T
Competence: Organize work, in terms of good time management and its ordering and planning.
Outcome of learning:
1991:T04.00 - Organize work, in terms of good time management and its ordering and planning.

Code: 1991:T09
Type: T
Competence: Using information technologies available and adapt to the new technological environments.
Outcome of learning:
1991:T09.00 - Using information technologies available and adapt to the new technological environments.

Course contents

Topic 1: The n-dimensional Euclidean space
Objective: To explain the n-dimensional Euclidean space and the basic operations on vectors (sum of vectors, multiplication by scalars, scalar product of two vectors), the notions of norm and Euclidean distance and the basic properties of sets (open, closed, bounded, compact and convex).

1.1. Vectors.
1.2. Sum of vectors.
1.3. Multiplication by scalars.
1.4. Scalar product of two vectors.
1.5. Properties of basic operations and their geometric interpretations.
1.6. Euclidean norm and its properties.
1.7. Euclidean distance and its properties.
1.8. Open and closed sets.
1.9. Bounded and compact sets.
1.10. Convex sets.

Case study: Budget constraints.
Textbook: Sections 12.2, 12.3 and 12.4.

Topic 2: Linear algebra
Objective: To introduce the basic notions of linear algebra: linear dependence and independence on vectors, matrices, determinants, inverse of a matrix and systems of linear equations.

2.1. Linear dependence and independence on vectors.
2.2. Matrices, determinants, inverse of a matrix and rank.
2.3. Rouché-Frobenius theorem.
2.4. Solving systems of linear equations: Cramer's rule and Gauss' system.

Case study: Calculation of determinants, inverse of a matrix and solutions of systems of linear equations.

Textbook: Chapters 12, 13 and 14.

Topic 3: Functions of several variables, geometric representation, and contour maps
Objective: Introduce the functions of several variables as a tool to describe relationships among economic variables. Geometric representation of functions of several variables as level curves and contour maps.

3.1. Functions of several variables.
3.2. Component functions.
3.3. Domain of the function.
3.4. Linear functions and associated matrices.
3.5. Level curves.
3.6. Contour maps.

Case study: Production functions, utility functions and wage functions. Cobb-Douglas functions, linear, quasi-linear, and Leontiev functions.

Textbook: Chapter 15.

Topic 4: Limit of a function at a point and continuity. Weierstrass theorem
Objective: Define the limit of a function at a point (and study their basic properties), the directional limits and continuous

functions (and study their properties). Present the Weierstrass theorem and provide the geometric properties of the constrained optima from the graphical representations of the constraint and the contour map of the function.

4.1. Limit of a function at a point.
4.2. Properties of the limit of a function at a point.
4.3. Directional limits.
4.4. Concept of a continuous function.
4.5. Continuous functions and directional limits.
4.6. Weierstrass theorem.

Case study: Obtention through geometric methods of the maxima of several functions of interest in economics and business.

Textbook: Sections 4.4, 6.1, 6.2 and 17.3.

Topic 5: Partial derivatives and differentiable functions
Objective: Define the directional and partial derivatives of functions of several variables, interpretation of their meaning. Introduce the gradient vector of a function at a point and its geometric interpretation as the growth of the function at the point. Define the Jacobian matrix. Define the differential of a function at a point and the differentiable functions. Introduce the chain rule, higher order derivatives, the Hessian matrix, and Schwartz's theorem.

5.1. Derivative of a function at a point in the direction of a unit vector.
5.2. Partial derivatives at one point.
5.3. Partial derivative functions.
5.4. Gradient of the function at one point. Geometrical interpretation and directional derivatives.
5.5. Jacobian of the function at one point.
5.6. Differentiable functions. Continuity of partial derivative functions.
5.7. Chain rule.
5.8. Higher order derivatives, Hessian matrix and Schwartz's theorem.

Case study: Obtention and geometric interpretation of the gradients of different functions of interest in economics and business.

Textbook: Chapters 15 and 16.

Topic 6: Implicit function theorem and inverse function theorem
Objective: Introduce the implicit function theorem and the inverse function theorem as instruments to obtain the linear approximation to the level curve of a function at a point and to perform comparative statics exercises respectively.

6.1. Implicit function theorem.
6.2. Inverse function theorem.
6.3. Applications and geometric intuition.

Case study: Obtaining the slope of the level curve of several functions of interest in economics and business. Exercises in comparative statics in several basic economic models.

Textbook: Section 16.3.

Topic 7: Unconstraint optimization
Objective: Introduce the fundamental techniques to optimize functions of several variables and introduce the concepts of concave and convex functions.

7.1. Quadratic forms.
7.2. Local and global optima.
7.3. First and second order conditions for local optima.
7.4. Global optima of concave and convex functions.

Case study: Identification of extreme points. Study of the concavity and convexity of several functions of interest in economics and business.

Textbook: Chapter 17.

Topic 8: Constrained optimization
Objective: Introduce the fundamental techniques for optimizing functions of several variables with equality constraints. Introduce the Lagrange theorem.

8.1. Maximization and minimization problems with equality constraints.
8.2. Constrained local optima. Lagrange's Theorem.
8.3. Constrained global optima of concave and convex functions.
8.4. Introduction to inequality constraints.

Case study: Solving optimization problems in economics and business.

Textbook: Chapter 18.

Methodology

To achieve the objectives of the course, the following taxonomy of activities will be used:

1. Theory classes where teachers develop the main concepts.
The objective of this activity is to present the fundamental notions of course, and to facilitate their learning through the analysis of examples illustrating the intuitions and economic applications.

2. Exercises sessions devoted to the resolution of problems.
This activity aims to discuss and answer any questions that students may have in solving the problem sets, and at the same time to correct mistakes. These sessions will also stimulate the participation of students presenting the solutions of the problem sets either orally or in written form.

3. Organized supervised activities, to apply the concepts studied to economic situations
The objective of this activity is to encourage the student to establish links between the mathematical tools and their use in economics. When possible, these sessions will be organized in small groups of students.

4. Problem solving by students
Each topic will have a list of associated problems that must be solved independently by students.
The objective of this activity is two-fold: on the one hand it aims at the reinforcement of the theoretical concepts and tools exposed in the theory sessions; on the other hand it aims at the acquisition of the skills required to solve exercises and problems.
We promote the cooperative resolution of problems in stable working groups of 3 or 4 students throughout the semester, to stimulate team work to overcome the difficulties that may arise to their components.

5. Tutorial attendance
Students have several hours where the teachers of the course may help them to resolve any doubts that may arise in the study of the course and in the solution of the problem sets. These sessions cannot be on-line, but face-to-face between the teacher and the students.

Learning activities and outcomes

Activities	Hours	ECTS	Learning outcomes
Type: Directed			
Theory classes	30		1991:E01.01, 1991:E01.02, 1991:E01.03, 1991:E01.04, 1991:E01.05, 1991:E01.06, 1991:E01.07, 1991:E39.01, 1991:E39.02
Type: Supervised			
Activity: Solving problem sets	15		1991:E01.01, 1991:E01.02, 1991:E01.03, 1991:E01.04, 1991:E01.05, 1991:E01.06, 1991:E01.07, 1991:E39.01, 1991:E39.02 1991:T01.00, 1991:T04.00, 1991:T09.00
Activity: Follow-up of homeworks	3		1991:E01.01, 1991:E01.02, 1991:E01.03, 1991:E01.04, 1991:E01.05, 1991:E01.06, 1991:E01.07, 1991:E39.01, 1991:E39.02 1991:T01.00, 1991:T04.00, 1991:T09.00
Tutorials	7		1991:E01.01, 1991:E01.02, 1991:E01.03, 1991:E01.04, 1991:E01.05, 1991:E01.06, 1991:E01.07, 1991:E39.01, 1991:E39.02
Type: Self learning			
Study	90		1991:E01.01, 1991:E01.02, 1991:E01.03, 1991:E01.04, 1991:E01.05, 1991:E01.06, 1991:E01.07, 1991:E39.01, 1991:E39.02 1991:T04.00, 1991:T09.00

Assessment

Activities and instruments used in the evaluation:
1. Individual written tests
Two individual written tests will be scheduled. The first one will take place in the sixth week (approximately); the second test will take place in week 12 (approximately). These tests will last 40 minutes and no support materials (calculators, class notes, etc.) will be allowed.

2. An individual comprehensive final exam.
This exam is designed to force students to make a last effort of learning necessary to consolidate the knowledge already acquired.
The exam will be comprehensive and all students have to take it. The exam will last up to two hours and no support materials

(calculators, class notes, etc.) will be allowed.
This double rating system aims at maximizing success in the learning process of the largest number of students.

3. Delivery of problem sets
Students may be required to solve at most two problem sets. This activity will be carried out as teamwork. A team will consist of a minimum of three persons and a maximum of six.

All students are required to take the exams and perform the other tasks in the dates set in the calendar of the course. In no case there can be an evaluation activity outside of the dates indicated.

Evaluation criteria:

(a) The final grade of the course obtains from the individual written tests and the individual final exam. Scoring each of the two activities with a value between 0 and 10, the final grade follows from the following weighted average:
FINAL GRADE = 50% (written activities) + 50% (final exam).
The weight of each of the two tests written will be the same (20%). If problem sets are delivered, their weight will be 10% of the final grade. In this case, the written tests will reduce their weight to 40%.

(b) The course will be considered as "passed" if the final grade is greater than or equal to 5.

(c) For those students who have obtained a final grade equal to or greater than 4 and less than 5, there will be a re-evaluation.
The re-evaluation consists in an individual written test scheduled in the last week of the semester. Those students with a right to attend the re-evaluation will obtain at most 1 point to be added to the student's grade. If the re-evaluated final grade reaches 5 points, the student will pass the course.

(d) A student is considered as "No-show" in the evaluation whenever he/she has not participated in any evaluation activities. Therefore, a student **participating in a single activity** cannot opt to be considered as "no-show" and a grade will be assigned.

Assessement activities

Activities	Weight	Hours	ECTS	Learning outcomes
Final exam	0.4	3		1991:E01.01, 1991:E01.02, 1991:E01.03, 1991:E01.04, 1991:E01.05, 1991:E01.06, 1991:E01.07, 1991:E39.01, 1991:E39.02
Written tests	0.6	2		1991:E01.01, 1991:E01.02, 1991:E01.03, 1991:E01.04, 1991:E01.05, 1991:E01.06, 1991:E01.07, 1991:E39.01, 1991:E39.02, 1991:T01.00, 1991:T04.00, 1991:T09.00

References

Main textbook:
Sydsaeter, K. and P.J. Hammond, 1995, *Mathematics for Economic Analysis*. London, Prentice Hall.

This is a textbook of great tradition and acceptance. In addition, this book also covers the subjects of Mathematics I. It is a complete and friendly text, including economic applications in all its chapters.

Complementary textbooks:

The textbooks listed below can be helpful to complement the explanations contained in the main textbook and also to students wishing to enlarge their knowledge.

Alegre, P., L. Jorba, F.J. Orti, G. Rodriguez, J.B. Saez, T. Sancho and A. Terceño, 2000, *Ejercicios Resueltos de Matemáticas Empresariales II*, Madrid, Alfacentauro.

Besada, M., F.J. García, M.A. Mirás and M.C. Vázquez, 2001, *Cálculo de varias variables. Cuestiones y ejercicios resueltos*, Madrid, Ed. Prentice Hall.

Chiang, A.C., 2005, *Fundamental Methods of Mathematical Economics,* McGraw-Hill.

Larson, R., R. Hostetler, and B. Edwards, 1994, *Calculus with Analytic Geometry*, Lexington, D.C. Heath.

Sydsaeter, K. and P.J. Hammond, 2002, *Essential Mathematics for Economic Analysis*. London, Prentice Hall.

Other complementary material will be uploaded in the webpage of the course.