Regions going global: The case of Brazilian trade in value added

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Abstract: In the last couple of decades, production processes have been characterized by their fragmentation, which crosses the borders of countries more and more. This coincides with the common viewpoint that products and services are now made in global value chains and that ‘trade in value added’ might be a better approach to capture a country’s role in international trade. The same applies at the regional level, and perhaps even to a larger extent. The present paper adopts a worldwide input-output framework to cover the internationally interdependent structure of today’s production processes. The empirical application combines a world input-output table covering 40 countries (and the rest of the world as a 41st country) with an inter-regional input-output table covering all Brazilian states, for the year 2008. Our results show that the average country trades approximately twice as much value added (as a share of the country’s value added) than Brazil. The paper also analyzes the trade in vlaue added between Brazilian states and finds that almost all states are net importers of value added from São Paulo.

Keywords: International trade, input-output analysis, regional economics

JEL Codes: F10, R15, R11

1. Introduction

In the recent past, production processes have increasingly become sliced up into ever smaller parts (or fragmented). Many of these parts are outsourced to specialized subcontractors that are more and more located in foreign countries (i.e. offshoring). This has led to an upsurge of trade in intermediate products, which corresponds to Baldwin's (2006) "second wave of global unbundling" where the location of the production of intermediate inputs differs from the location of the production of the final products.[1] Theories to explain the relocation of the production of intermediate inputs to other countries have been developed e.g. by Grossman and Rossi-Hansberg (2008) and Costinot *et al*. (2013).

Today's products and services are no longer produced within a single country. Instead, they are made in global supply chains, or global value chains. That is, countries import intermediate goods and raw materials, to which they add one or more layers of value after which they sell the product (often to a foreign producer who adds the next layer). Standard trade figures that measure the value of imports and exports do not reflect any more what is really happening. This has also attracted the attention of policy makers. For example, according to EU Commissioner for Trade Karel De Gucht: [2] "The country that exports the final product is artificially credited with having created all of its value, even if in reality it only assembled ready-made parts. … This is a bit like the final runner in a relay team getting a gold medal while his teammates get silver and bronze. It doesn't take account of the fact that the final result is the product of a joint effort." Recently, Pascal Lamy (Director-General of the WTO) launched jointly with the OECD, the "made in the world" initiative and proposed "trade in value added" as a better approach for the measurement for international trade (see OECD-WTO, 2012).[3]

The same applies at the regional level, and perhaps even to a larger extent. Due to locational advantages (e.g. the presence of a seaport and/or an airport), one region is typically responsible for most of the imports and exports of a country. Even if the production of some export goods takes place entirely within the country, it is likely that other regions than the exporting region also have contributed to the value of the exports.

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[1] The first wave of global unbundling refers to the separation of the location of consumption and the location of production were separated, which led to increased trade in final products.
[2] Available at: http://trade.ec.europa.eu/doclib/docs/2012/april/tradoc_149337.pdf.
[3] The OECD-WTO database with trade in values added (TiVA) is available at: http://stats.oecd.org/Index.aspx?DataSetCode=TIVA_OECD_WTO.

For example by supplying intermediate inputs and raw materials that go into the final product that is sold abroad. Next to international fragmentation, also interregional fragmentation plays a role when focusing on regions.

In the Brazilian case, an example of such a regional value chain is given by the exports of the Volkswagen Gol to foreign countries in Latin America. The Anchieta plant, located in the state of São Paulo, is responsible for the assembling of the Gol model, which is subsequently exported through the Port of Santos, also in the state of São Paulo. For that assembling process, the Anchieta Plant uses, among many other intermediate inputs, flat carbon steel products from the Companhia Siderurgica Nacional's plant in the state of Rio de Janeiro. In its turn, that steel plant requires, for example, iron ores from the state of Minas Gerais. Therefore, the exports of the Gol model generate value added not only in the state of São Paulo, but also in Rio de Janeiro, and in Minas Gerais.

The present paper analyzes the role of Brazilian regions in the global value chain. We will do this by answering the question: How much value added generated in, for example, the state of Minas Gerais is embodied in the "consumption" bundle of, for example, Mexico? This gives us the export of value added from Minas Gerais to Mexico. It should be stressed that the "consumption" bundle includes household consumption, government expenditures, gross fixed capital formation, and changes in inventories. It should also be stressed that the Mexican consumption bundle includes imported goods from other countries, such as US, or other Brazilian states, such as São Paulo. Indirectly, these goods may include value added from Minas Gerais. This is the case in the example above of the Volswagen Gol. So, in principle, it may be the case that Minas Gerais does not export to Mexico but that some of its value added is still embodied in Mexican consumption (for example through US exports or São Paulo exports that are produced using intermediate products from Minas Gerais). In the same fashion, we will also calculate the import of Mexican value added by Minas Gerais.

Whereas the methodology to calculate the trade in value added is well-known, the availability of data used to be a problem.[4] In recent years, however, several groups of researchers have developed so-called world input-output tables (WIOTs). These are

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[4] As a matter of fact, the calculation of trade in value added is very similar to the calculation of trade in emissions, the methodology of which is known for quite some time (see, e.g. Serrano and Dietzenbacher, 2010, for a methodological overview). With respect to data availability, a rare exception is the series of intercountry IO tables for a limited set of European countries for 1965, 1970, 1975, 1980, 1985 (Van der Linden and Oosterhaven, 1995, and Van der Linden, 1999). These tables are available at: http://www.rug.nl/research/reg/research/irios/irios-tables.

interregional Isard-type input-output tables with countries instead of regions. The now available WIOTs typically include a large number of individual countries and a “country” that reflects the rest of the world.[5] The present paper combines the WIOT from the World Input-Output Database (WIOD) project, which includes Brazil as one of its countries, with an inter-regional input-output table (IRIOT) for Brazil.[6]

The next section discusses the methodology that was used to combine the two datasets and Section 3 reflects on the compatibility of the datasets. The results are presented and discussed in Section 4 and Section 5 concludes.

2. Methodology

In our analysis, we will combine a world input-output table (WIOT) with an inter-regional input-output table (IRIOT). For the empirical application, we will use the WIOT for 2008 that was constructed in the WIOD project (see Dietzenbacher *et al.*, 2013). It is a full inter-country input-output (IO) table covering 40 countries and the rest of the world as a 41st country. One of the countries included is Brazil. The IRIOT for 2008 is for Brazil and covers the 27 Brazilian states. Both the WIOT and the IRIOT were aggregated to 28 compatible industries. The classification of regions (i.e. countries and Brazilian states) and industries can be found in Appendix A.

We are aware of two other papers that have split one of the countries in a WIOT into a number of regions. Cherubini and Los (2012) used WIOD data and split Italy into four regions and Feng et al. (2013) used GTAP-MRIO data and split China into 30 provinces. Constructing such a combined country-state table for Brazil would, in principle, follow two routes. On the one hand, one might take the whole of Brazil in the WOT as given and adapt the IRIOT accordingly. On the other hand, one might take the IRIOT as given and adpat the WIOT accordingly. In this paper we follow an entirely different approach and combine the “best of both worlds” by working with the input coefficients rather than the deliveries themselves.

To outline the methodology, we will use a much smaller case as an example. Without loss of generality we will employ a WIOT for a world that consists of three

[5] See Tukker and Dietzenbacher (2013) for an overview. Examples are Eora (Lenzen et al., 2012, 2013), EXIOBASE (Tukker et al., 2009, 2013), GTAP-MRIO (Peters et al., 2011; Andrew and Peters, 2013), WIOD (Dietzenbacher et al., 2013), the OECD database (Nakano et al., 2009) and GRAM (Bruckner et al., 2012; Wiebe et al., 2012).
[6] The full database from the WIOD project (including a time series of WIOTs) is publicly and free of charge available at: http://www.wiod.org/database/index.htm.

countries (R, S, T). For country T we have an IRIOT, distinguishing between two regions (east E and west W). The WIOT is given in Figure 1 and the IRIOT in Figure 2.

Figure 1. The World Input-Output Table

	Intermediate use			Final use			Gross
	in R	in S	in T	in R	in S	in T	Output
Product flows from							
country R	$\mathbf{Z}^{RR}$	$\mathbf{Z}^{RS}$	$\mathbf{Z}^{RT}$	$\mathbf{c}^{RR}$	$\mathbf{c}^{RS}$	$\mathbf{c}^{RT}$	$\mathbf{x}^{R}$
country S	$\mathbf{Z}^{SR}$	$\mathbf{Z}^{SS}$	$\mathbf{Z}^{ST}$	$\mathbf{c}^{SR}$	$\mathbf{c}^{SS}$	$\mathbf{c}^{ST}$	$\mathbf{x}^{S}$
country T	$\mathbf{Z}^{TR}$	$\mathbf{Z}^{TS}$	$\mathbf{Z}^{TT}$	$\mathbf{c}^{TR}$	$\mathbf{c}^{TS}$	$\mathbf{c}^{TT}$	$\mathbf{x}^{T}$
Value added	$(\mathbf{v}^{R})'$	$(\mathbf{v}^{S})'$	$(\mathbf{v}^{T})'$				
Total inputs	$(\mathbf{x}^{R})'$	$(\mathbf{x}^{S})'$	$(\mathbf{x}^{T})'$				

For example, $\mathbf{Z}^{RS}$ is an n×n matrix and its typical element z_{ij}^{RS} indicates the delivery of intermediate inputs from industry i in country R to industry j in country S.[7] Note that i, j = 1, …, n where n is the number of industries. In case $R \neq S$, the matrix $\mathbf{Z}^{RS}$ indicates the exports of country R to industries in country S. $\mathbf{c}^{RS}$ is an n-element vector and its typical element c_i^{RS} indicates the final use (also termed final demand) in country S of goods and services produced by industry i in country R. Final use covers household and government consumption, consumption of non-profit organizations, gross fixed capital formation, and changes in inventories. Again, if $R \neq S$, $\mathbf{c}^{RS}$ indicates the exports of country R to final users in country S. $\mathbf{x}^{R}$ is an n-element vector with its typical element x_i^R indicating the gross output of industry i in country R. Finally, $\mathbf{v}^{R}$ is an n-element vector and its typical element v_i^R gives the value added in industry i of country R.

[7] Matrices are given in bold capital letters (e.g. $\mathbf{Z}^{RS}$), vectors are given in bold lower case letters (e.g. $\mathbf{x}^{R}$), and scalars (including matrix or vector elements) are given in italicized lower case letters (e.g. x_i^R). Vectors are columns by definition, row vectors are obtained by transposition, which is indicated by a prime (e.g. $(\mathbf{v}^{R})'$). We will use a circumflex or "hat" to indicate a diagonal matrix (e.g. $\hat{\mathbf{x}}^{R}$) with the elements of the corresponding vector (i.e. $\mathbf{x}^{R}$) on the main diagonal and all other elements equal to zero.

Figure 2. The Inter-regional Input-Output Table

	Intermediate use in *E*	Intermediate use in *W*	Final use in *E*	Final use in *W*	Exports	Gross output
Product flows from						
region *E*	$\mathbf{Z}^{EE}$	$\mathbf{Z}^{EW}$	$\mathbf{c}^{EE}$	$\mathbf{c}^{EW}$	$\mathbf{e}^{E}$	$\mathbf{x}^{E}$
region *W*	$\mathbf{Z}^{WE}$	$\mathbf{Z}^{WW}$	$\mathbf{c}^{WE}$	$\mathbf{c}^{WW}$	$\mathbf{e}^{W}$	$\mathbf{x}^{W}$
Imports	$(\mathbf{m}^{E})'$	$(\mathbf{m}^{W})'$	h^{E}	h^{W}		
Value added	$(\mathbf{v}^{E})'$	$(\mathbf{v}^{W})'$				
Total inputs	$(\mathbf{x}^{E})'$	$(\mathbf{x}^{W})'$				

The interpretation of the matrices **Z** and the vectors **c**, **x** and **v** for the IRIOT in Figure 2 is similar to the interpretation in the case of the WIOT in Figure 1. For example, z_{ij}^{EW} gives the flows of goods and services from industry *i* in region *E* to industry *j* in region *W*, c_i^{EW} indicates the delivery by industry *i* in region *E* to final users in region *W*, and x_i^E and v_i^E give the gross output and value added in industry *i* in region *E*. In addition, $\mathbf{e}^E$ is an *n*-element export vector and its typical element e_i^E gives the total exports by industry *i* in region *E*. Note that no information is available for the distribution of the exports over destinations (i.e. countries, as well as their industries and final users). The Brazilian IRIOT provides information for the total imports by each industry, i.e. without making a distinction between the countries of origin. For region *E* for example, $\mathbf{m}^E$ is the *n*-element import vector and its typical element m_i^E indicates the total imports by industry *i* in region *E*. Finally, the scalar h^E gives the total imports purchased by final users in region *E*. Again, it is not known how much each country of origin delivers to final users in *E*, just the total of these deliveries is known. It should be stressed that other import information is available (and will be used and discussed later) that is not indicated in Figure 2.

When combining the information from the WIOT and the IRIOT, we might start by constructing an enlarged input-output table. The information for country *T* in the WIOT is then to be replaced by the information for regions *E* and *W* from the IRIOT. One of the problems, however, is that the information for country *T* in the WIOT is not entirely consistent with the summation of the information for regions *E* and *W*. Instead, we will therefore carry out the analysis by using the WIOT for calculations at the country level and using the IRIOT for calculations at the regional level, and iterating back and forth.

The question that we will start with is: What are the output levels (in countries *R* and *S*, and regions *E* and *W*) necessary to satisfy an arbitrary final demand vector? We split this question and consider the outputs necessary for arbitrary final demand vectors $\mathbf{y}^R$ and $\mathbf{y}^S$ first. We use the round-by-round approach that is common in IO analysis. In the first round, the final demands need to be produced themselves. That is, $\mathbf{y}^R$ in country *R* and $\mathbf{y}^S$ in country *S*. In the second round we need to calculate how much inputs are required (i.e. the direct inputs). Let the input matrices be defined as usual. That is, for example, $\mathbf{A}^{RS} = \mathbf{Z}^{RS}(\hat{\mathbf{x}}^S)^{-1}$ with its typical element $a_{ij}^{RS} = a_{ij}^{RS}/x_j^S$ indicating the input from industry *i* in country *R* that goes to (and is measured per unit of output of) industry *j* in country *S*. The direct inputs amount to $\mathbf{A}^{RR}\mathbf{y}^R + \mathbf{A}^{RS}\mathbf{y}^S$ in country *R* and to $\mathbf{A}^{SR}\mathbf{y}^R + \mathbf{A}^{SS}\mathbf{y}^S$ in country *S*.

From the WIOT, it follows that the direct inputs required from country *T* amount to $\mathbf{A}^{TR}\mathbf{y}^R + \mathbf{A}^{TS}\mathbf{y}^S$. Note that these are exports of country *T* to country *R* ($\mathbf{A}^{TR}\mathbf{y}^R$) and to country *S* ($\mathbf{A}^{TS}\mathbf{y}^S$). For our analysis we would need to know the exports of region *E* to country *R* (i.e. $\mathbf{A}^{ER}\mathbf{y}^R$) and from region *W* to country *R* (i.e. $\mathbf{A}^{WR}\mathbf{y}^R$), but the input matrices $\mathbf{A}^{ER}$ and $\mathbf{A}^{WR}$ are unknown. Based on information that is not listed in the Brazilian IRIOT, by deriving export shares, we estimate how much of the exports to *R*, for example, originates from region *E* and how much from *W*. Let the vector $\boldsymbol{\sigma}^{ER}$ denote the vector of export shares. Its elements are defined as follows.

$$\sigma_i^{ER} = e_i^{ER}/(e_i^{ER} + e_i^{WR})$$

which indicates the share of the exports of product *i* to country *R* that stems from region *E*. The shares from region *W* are defined similarly and the shares add to one (i.e. $\sigma_i^{ER} + \sigma_i^{WR} = 1$).

We assume that the export shares apply irrespective of the industry of destination in country *R*. Our estimate (indicated by a tilde) for the input coefficients then yields $\tilde{a}_{ij}^{ER} = \sigma_i^{ER} a_{ij}^{TR}$. The direct inputs from region *E* (which are exported to country *R*) then become $\tilde{\mathbf{A}}^{ER}\mathbf{y}^R = \hat{\boldsymbol{\sigma}}^{ER}\mathbf{A}^{TR}\mathbf{y}^R$, and those from region *W* are given by $\tilde{\mathbf{A}}^{WR}\mathbf{y}^R = \hat{\boldsymbol{\sigma}}^{WR}\mathbf{A}^{TR}\mathbf{y}^R$.

In the same fashion, the exports of country *T* to country *S* ($\mathbf{A}^{TS}\mathbf{y}^{S}$) in this first round must be split into the exports from region *E* and those from region *W*, which are estimated using the export shares for exports to *S*. That is, using $\sigma_i^{ES} = e_i^{ES}/(e_i^{ES} + e_i^{WS})$ and $\sigma_i^{WS} = e_i^{WS}/(e_i^{ES} + e_i^{WS})$, the exports from region *E* and from region *W* are estimated as $\widetilde{\mathbf{A}}^{ES}\mathbf{y}^{S} = \widehat{\boldsymbol{\sigma}}^{ES}\mathbf{A}^{TS}\mathbf{y}^{S}$ and $\widetilde{\mathbf{A}}^{WS}\mathbf{y}^{S} = \widehat{\boldsymbol{\sigma}}^{WS}\mathbf{A}^{TS}\mathbf{y}^{S}$. The direct inputs necessary for the final demand vectors $\mathbf{y}^{R}$ and $\mathbf{y}^{S}$ are given by

$$\begin{bmatrix} \mathbf{A}^{RR} & \mathbf{A}^{RS} \\ \mathbf{A}^{SR} & \mathbf{A}^{SS} \\ \widetilde{\mathbf{A}}^{ER} & \widetilde{\mathbf{A}}^{ES} \\ \widetilde{\mathbf{A}}^{WR} & \widetilde{\mathbf{A}}^{WS} \end{bmatrix} \begin{pmatrix} \mathbf{y}^{R} \\ \mathbf{y}^{S} \end{pmatrix} = \begin{bmatrix} \mathbf{A}^{RR} & \mathbf{A}^{RS} \\ \mathbf{A}^{SR} & \mathbf{A}^{SS} \\ \widehat{\boldsymbol{\sigma}}^{ER}\mathbf{A}^{TR} & \widehat{\boldsymbol{\sigma}}^{ES}\mathbf{A}^{TS} \\ \widehat{\boldsymbol{\sigma}}^{WR}\mathbf{A}^{TR} & \widehat{\boldsymbol{\sigma}}^{WS}\mathbf{A}^{TS} \end{bmatrix} \begin{pmatrix} \mathbf{y}^{R} \\ \mathbf{y}^{S} \end{pmatrix} \tag{1}$$

The second question is similar to the first one, namely what are the output levels (in countries *R* and *S*, and regions *E* and *W*) necessary to satisfy arbitrary final demand vectors $\mathbf{y}^{E}$ and $\mathbf{y}^{W}$. In the first round these final demands are produced themselves. For the direct inputs, the input matrices are obtained from the IRIOT. For example, we have $\mathbf{A}^{EW} = \mathbf{Z}^{EW}(\hat{\mathbf{x}}^{W})^{-1}$ with its typical element $a_{ij}^{EW} = a_{ij}^{EW}/x_j^{W}$ indicating the input from industry *i* in region *E* that goes to (and is measured per unit of output of) industry *j* in region *W*. The direct regional inputs amount to $\mathbf{A}^{EE}\mathbf{y}^{E} + \mathbf{A}^{EW}\mathbf{y}^{W}$ in region *E* and to $\mathbf{A}^{WE}\mathbf{y}^{E} + \mathbf{A}^{WW}\mathbf{y}^{W}$ in region *W*.

To calculate the direct (imported) inputs from country *R*, we would need to have the input coefficients a_{ij}^{RE} and a_{ij}^{RW}. Because they are not available, they need to be estimated from the information given in Figures 1 and 2. From the WIOT, we know the value of a_{ij}^{RT}. By definition, this should be equal to $(z_{ij}^{RE} + z_{ij}^{RW})/(x_j^{E} + x_j^{W})$. Since information for z_{ij}^{RE} and z_{ij}^{RW} is lacking, we first estimate their sum by

$$\tilde{z}_{ij}^{RE} + \tilde{z}_{ij}^{RW} = a_{ij}^{RT}(x_j^{E} + x_j^{W})$$

Next we will use the average import shares that are obtained from information that is not listed in the Brazilian IRIOT. That is, for region *E* it is known for each product *i* (i.e. the typical product or service produced by industry *i*) how much is imported from country *R* and how much from country *S*. The same applies to the imports by region *W*. Note that no information is available with respect to the distribution over the (intermediate and final) users in the importing region. Let the vector $\boldsymbol{\lambda}^{RE}$ denote the vector of import shares, the elements of which are defined as follows

$$\lambda_i^{RE} = \frac{\text{total imports of product } i \text{ by region } E \text{ from country } R}{\text{total imports of product } i \text{ (by regions } E \text{ and } W\text{) from country } R}$$

A similar definition holds for λ_i^{RW} and we have that $\lambda_i^{RE} + \lambda_i^{RW} = 1$. The shares for the imports from country *S* are defined similarly and add to one again (i.e. $\lambda_i^{SE} + \lambda_i^{SW} = 1$). We assume that these average import shares apply to each industry *j* of destination. That is,

$$\tilde{z}_{ij}^{RE} = \lambda_i^{RE}(\tilde{z}_{ij}^{RE} + \tilde{z}_{ij}^{RW}) = \lambda_i^{RE} a_{ij}^{RT}(x_j^E + x_j^W).$$

Finally, let the vector $\boldsymbol{\mu}^E$ denote the vector of output shares in region *E* with its elements defined as

$$\mu_i^E = x_i^E/(x_i^E + x_i^W)$$

and a similar definition for the output shares of region *W*. This yields for the estimated input coefficients

$$\tilde{a}_{ij}^{RE} = \frac{\tilde{z}_{ij}^{RE}}{x_j^E} = \frac{\lambda_i^{RE} a_{ij}^{RT}(x_j^E + x_j^W)}{x_j^E} = \frac{\lambda_i^{RE} a_{ij}^{RT}}{\mu_j^E}\,.$$

In matrix notation we have $\tilde{\mathbf{A}}^{RE} = \hat{\boldsymbol{\lambda}}^{RE}\mathbf{A}^{RT}(\hat{\boldsymbol{\mu}}^E)^{-1}$ for region *E* and, similarly, for region *W* we have $\tilde{\mathbf{A}}^{RW} = \hat{\boldsymbol{\lambda}}^{RW}\mathbf{A}^{RT}(\hat{\boldsymbol{\mu}}^W)^{-1}$.

The direct inputs in country *R* necessary to satisfy the final demand vectors $\mathbf{y}^E$ and $\mathbf{y}^W$ are then given by $\tilde{\mathbf{A}}^{RE}\mathbf{y}^E$ $+\tilde{\mathbf{A}}^{RW}\mathbf{y}^W$. Those in country *S* are given by $\tilde{\mathbf{A}}^{SE}\mathbf{y}^E$ $+\tilde{\mathbf{A}}^{SW}\mathbf{y}^W$. Together with the regional direct inputs, this yields

$$\begin{bmatrix} \tilde{\mathbf{A}}^{RE} & \tilde{\mathbf{A}}^{RW} \\ \tilde{\mathbf{A}}^{SE} & \tilde{\mathbf{A}}^{SW} \\ \mathbf{A}^{EE} & \mathbf{A}^{EW} \\ \mathbf{A}^{WE} & \mathbf{A}^{WW} \end{bmatrix} \begin{pmatrix} \mathbf{y}^E \\ \mathbf{y}^W \end{pmatrix} = \begin{bmatrix} \hat{\boldsymbol{\lambda}}^{RE}\mathbf{A}^{RT}(\hat{\boldsymbol{\mu}}^E)^{-1} & \hat{\boldsymbol{\lambda}}^{RW}\mathbf{A}^{RT}(\hat{\boldsymbol{\mu}}^W)^{-1} \\ \hat{\boldsymbol{\lambda}}^{SE}\mathbf{A}^{ST}(\hat{\boldsymbol{\mu}}^E)^{-1} & \hat{\boldsymbol{\lambda}}^{SW}\mathbf{A}^{ST}(\hat{\boldsymbol{\mu}}^W)^{-1} \\ \mathbf{A}^{EE} & \mathbf{A}^{EW} \\ \mathbf{A}^{WE} & \mathbf{A}^{WW} \end{bmatrix} \begin{pmatrix} \mathbf{y}^E \\ \mathbf{y}^W \end{pmatrix} \qquad (2)$$

Finally, combining equations (1) and (2) gives us the direct inputs (in countries *R* and *S*, and regions *E* and *W*) that are necessary for an arbitrary final demand vector. They are given by

$$\begin{bmatrix} \mathbf{A}^{RR} & \mathbf{A}^{RS} & \widetilde{\mathbf{A}}^{RE} & \widetilde{\mathbf{A}}^{RW} \\ \mathbf{A}^{SR} & \mathbf{A}^{SS} & \widetilde{\mathbf{A}}^{SE} & \widetilde{\mathbf{A}}^{SW} \\ \widetilde{\mathbf{A}}^{ER} & \widetilde{\mathbf{A}}^{ES} & \mathbf{A}^{EE} & \mathbf{A}^{EW} \\ \widetilde{\mathbf{A}}^{WR} & \widetilde{\mathbf{A}}^{WS} & \mathbf{A}^{WE} & \mathbf{A}^{WW} \end{bmatrix} \begin{pmatrix} \mathbf{y}^R \\ \mathbf{y}^S \\ \mathbf{y}^E \\ \mathbf{y}^W \end{pmatrix} =$$

$$= \begin{bmatrix} \mathbf{A}^{RR} & \mathbf{A}^{RS} & \hat{\boldsymbol{\lambda}}^{RE}\mathbf{A}^{RT}(\hat{\boldsymbol{\mu}}^{E})^{-1} & \hat{\boldsymbol{\lambda}}^{RW}\mathbf{A}^{RT}(\hat{\boldsymbol{\mu}}^{W})^{-1} \\ \mathbf{A}^{SR} & \mathbf{A}^{SS} & \hat{\boldsymbol{\lambda}}^{SE}\mathbf{A}^{ST}(\hat{\boldsymbol{\mu}}^{E})^{-1} & \hat{\boldsymbol{\lambda}}^{SW}\mathbf{A}^{ST}(\hat{\boldsymbol{\mu}}^{W})^{-1} \\ \hat{\boldsymbol{\sigma}}^{ER}\mathbf{A}^{TR} & \hat{\boldsymbol{\sigma}}^{ES}\mathbf{A}^{TS} & \mathbf{A}^{EE} & \mathbf{A}^{EW} \\ \hat{\boldsymbol{\sigma}}^{WR}\mathbf{A}^{TR} & \hat{\boldsymbol{\sigma}}^{WS}\mathbf{A}^{TS} & \mathbf{A}^{WE} & \mathbf{A}^{WW} \end{bmatrix} \begin{pmatrix} \mathbf{y}^R \\ \mathbf{y}^S \\ \mathbf{y}^E \\ \mathbf{y}^W \end{pmatrix} \tag{3}$$

Let us write these direct inputs in condensed form as $\mathbf{Ay}$. The production of the direct inputs requires further inputs to the amount of $\mathbf{A}^2\mathbf{y}$, and so forth. Together with the initial outputs ($\mathbf{y}$), this yields $(\mathbf{I} + \mathbf{A} + \mathbf{A}^2 + \cdots)\mathbf{y} = (\mathbf{I} - \mathbf{A})^{-1}\mathbf{y} = \mathbf{Ly}$, where $\mathbf{L}$ denotes the Leontief inverse which can be partitioned in the same way as $\mathbf{A}$.

In our application, we are not so much interested in the output levels (necessary for an arbitrary final demand vector) but in the value added. Define the value added coefficients for country *R* as $g_i^R = v_i^R / x_i^R$, indicating the value added per unit of output. In matrix notation this becomes $(\mathbf{g}^R)' = (\mathbf{v}^R)'(\hat{\mathbf{x}}^R)^{-1}$. The total values added created in country *R*, in country *S*, in region *E*, and in region *W*, necessary for the final demand vector $\mathbf{y}$ are given by the four elements of the vector

$$\begin{bmatrix} (\mathbf{g}^R)' & 0 & 0 & 0 \\ 0 & (\mathbf{g}^S)' & 0 & 0 \\ 0 & 0 & (\mathbf{g}^E)' & 0 \\ 0 & 0 & 0 & (\mathbf{g}^W)' \end{bmatrix} \begin{bmatrix} \mathbf{L}^{RR} & \mathbf{L}^{RS} & \mathbf{L}^{RE} & \mathbf{L}^{RW} \\ \mathbf{L}^{SR} & \mathbf{L}^{SS} & \mathbf{L}^{SE} & \mathbf{L}^{SW} \\ \mathbf{L}^{ER} & \mathbf{L}^{ES} & \mathbf{L}^{EE} & \mathbf{L}^{EW} \\ \mathbf{L}^{WR} & \mathbf{L}^{WS} & \mathbf{L}^{WE} & \mathbf{L}^{WW} \end{bmatrix} \begin{pmatrix} \mathbf{y}^R \\ \mathbf{y}^S \\ \mathbf{y}^E \\ \mathbf{y}^W \end{pmatrix} \tag{4}$$

The central question in our application is how much value added that is generated in region *E* (or *W*) is contained in the final use of, for example, country *R*? This is the export of value added from region *E* (or *W*) to country *R*. We apply equation (4) and now take the final demand vector of country *R* instead of an arbitrary final demand vector. Final users in country *R* demand $\mathbf{c}^{RR}$ of domestically produced goods and services, import $\mathbf{c}^{SR}$ from country *S*, and $\mathbf{c}^{TR}$ from country *T*. Like we did with the input matrices, we use the export shares of regions *E* and *W* to split $\mathbf{c}^{TR}$. That is, $\hat{\boldsymbol{\sigma}}^{ER}\mathbf{c}^{TR}$ gives the exports from region *E* to final users in country *R* and $\hat{\boldsymbol{\sigma}}^{WR}\mathbf{c}^{TR}$ gives the exports from region *W*. This yields

$$\begin{bmatrix} (\mathbf{g}^R)' & 0 & 0 & 0 \\ 0 & (\mathbf{g}^S)' & 0 & 0 \\ 0 & 0 & (\mathbf{g}^E)' & 0 \\ 0 & 0 & 0 & (\mathbf{g}^W)' \end{bmatrix} \begin{bmatrix} \mathbf{L}^{RR} & \mathbf{L}^{RS} & \mathbf{L}^{RE} & \mathbf{L}^{RW} \\ \mathbf{L}^{SR} & \mathbf{L}^{SS} & \mathbf{L}^{SE} & \mathbf{L}^{SW} \\ \mathbf{L}^{ER} & \mathbf{L}^{ES} & \mathbf{L}^{EE} & \mathbf{L}^{EW} \\ \mathbf{L}^{WR} & \mathbf{L}^{WS} & \mathbf{L}^{WE} & \mathbf{L}^{WW} \end{bmatrix} \begin{pmatrix} \mathbf{c}^{RR} \\ \mathbf{c}^{SR} \\ \hat{\boldsymbol{\sigma}}^{ER}\mathbf{c}^{TR} \\ \hat{\boldsymbol{\sigma}}^{WR}\mathbf{c}^{TR} \end{pmatrix} \tag{5}$$

The third element of this vector gives the value added of region *E* that is embodied in the final use of country *R* and the fourth element gives the final demand of region *W*.

In the same fashion are we also interested in the value added generated in country *R* that is imported by region *E*. That is, in answering the question how much of country *R*'s value added is embodied in the final use of region *E*. The final demand for goods and services produced in the own region are given by $\mathbf{c}^{EE}$, for imports from region *W* by $\mathbf{c}^{WE}$. The imports by region *E*'s final users of products from country *R* are unknown. Like we did with the input matrices for the imports, the imports for final use in region *E* are estimated using import shares which yields $\hat{\boldsymbol{\lambda}}^{RE}\mathbf{c}^{RT}$. The same procedure is followed for the imports by final users from country *S*. This yields

$$\begin{bmatrix} (\mathbf{g}^R)' & 0 & 0 & 0 \\ 0 & (\mathbf{g}^S)' & 0 & 0 \\ 0 & 0 & (\mathbf{g}^E)' & 0 \\ 0 & 0 & 0 & (\mathbf{g}^W)' \end{bmatrix} \begin{bmatrix} \mathbf{L}^{RR} & \mathbf{L}^{RS} & \mathbf{L}^{RE} & \mathbf{L}^{RW} \\ \mathbf{L}^{SR} & \mathbf{L}^{SS} & \mathbf{L}^{SE} & \mathbf{L}^{SW} \\ \mathbf{L}^{ER} & \mathbf{L}^{ES} & \mathbf{L}^{EE} & \mathbf{L}^{EW} \\ \mathbf{L}^{WR} & \mathbf{L}^{WS} & \mathbf{L}^{WE} & \mathbf{L}^{WW} \end{bmatrix} \begin{pmatrix} \hat{\boldsymbol{\lambda}}^{RE}\mathbf{c}^{RT} \\ \hat{\boldsymbol{\lambda}}^{SE}\mathbf{c}^{ST} \\ \mathbf{c}^{EE} \\ \mathbf{c}^{WE} \end{pmatrix} \tag{6}$$

The first element of this vector gives the value added generated in country *R* that is embodied in the final use in region *E* and the second element gives the import of value added of country *S* by final users in region *E*.

3. Testing the quality of the estimation

In this section we check the quality of the estimation (or construction) of input coefficients or, alternatively, the compatibility of the two datasets. Consider the constructed input matrix, that is,

$$\begin{bmatrix} \mathbf{A}^{RR} & \mathbf{A}^{RS} & \tilde{\mathbf{A}}^{RE} & \tilde{\mathbf{A}}^{RW} \\ \mathbf{A}^{SR} & \mathbf{A}^{SS} & \tilde{\mathbf{A}}^{SE} & \tilde{\mathbf{A}}^{SW} \\ \tilde{\mathbf{A}}^{ER} & \tilde{\mathbf{A}}^{ES} & \mathbf{A}^{EE} & \mathbf{A}^{EW} \\ \tilde{\mathbf{A}}^{WR} & \tilde{\mathbf{A}}^{WS} & \mathbf{A}^{WE} & \mathbf{A}^{WW} \end{bmatrix} \tag{7}$$

We will carry out two sets of comparisons. First, aggregating over all the 27 Brazilian states (i.e. regions *E* and *W*, together making up country *T* = Brazil) yields

$$\begin{bmatrix} \mathbf{A}^{RR} & \mathbf{A}^{RS} & \tilde{\mathbf{A}}^{RT} \\ \mathbf{A}^{SR} & \mathbf{A}^{SS} & \tilde{\mathbf{A}}^{ST} \\ \mathbf{A}^{TR} & \mathbf{A}^{TS} & \tilde{\mathbf{A}}^{TT} \end{bmatrix}$$

(Appendix B details how the matrices $\tilde{\mathbf{A}}^{TT}$, $\tilde{\mathbf{A}}^{RT}$ and $\tilde{\mathbf{A}}^{ST}$ have been obtained.) The question is how this compares to the input coefficients matrix from the WIOT. By construction, differences only exist in the column with input matrices for Brazil. We separately compare the domestic input coefficients (i.e. $\tilde{\mathbf{A}}^{TT}$ and $\mathbf{A}^{TT}$, with *T* for Brazil) and the whole set of import coefficients matrices (i.e. $\tilde{\mathbf{A}}^{iT}$ and $\mathbf{A}^{iT}$, for all $i \neq T$).

The comparison will be based on the weighted absolute percentage error (WAPE), which has been frequently applied in the literature (Oosterhaven *et al*., 2008; Jiang *et al*., 2010). It attributes larger weights to errors in larger cells. For example the WAPE for the domestic input coefficients is defined as:

$$WAPE = 100 \times \frac{\sum_i \sum_j a_{ij}^{TT} \left| \frac{\tilde{a}_{ij}^{TT} - a_{ij}^{TT}}{a_{ij}^{TT}} \right|}{\sum_i \sum_j a_{ij}^{TT}} = 100 \times \frac{\sum_i \sum_j \left| \tilde{a}_{ij}^{TT} - a_{ij}^{TT} \right|}{\sum_i \sum_j a_{ij}^{TT}}$$

The second comparison aggregates the matrix in (7) over all 40 countries and compares it with the Brazilian IRIOT. Aggregation (see Appendix B for the details) yields estimates for the vectors of state exports (i.e. $\tilde{\mathbf{e}}^{ER}$, $\tilde{\mathbf{e}}^{ES}$, $\tilde{\mathbf{e}}^{WR}$, and $\tilde{\mathbf{e}}^{WS}$) and for the vectors of state import coefficients (i.e. estimates of $(\mathbf{m}^{E})'(\hat{\mathbf{x}}^{E})^{-1}$ and $(\mathbf{m}^{W})'(\hat{\mathbf{x}}^{W})^{-1}$).

The results for the WAPE in the two comparisons are presented in Table 1. Observe that in the first comparison, the WAPE for the domestic input coefficients is considerably larger than the WAPE for the import coefficients. Note that the estimated domestic input coefficients $\tilde{\mathbf{A}}^{TT}$ are largely based on the IRIOT, whereas the estimated import coefficients $\tilde{\mathbf{A}}^{iT}$ are largely based on the import coefficients from the WIOT. The difference in the WAPEs therefore points at inconsistencies between the datasets.

Table 1. WAPEs of the estimations (%)

		28 sectors	26 sectors
		WAPE	
Comparison 1: aggregate states and compare with Brazil in WIOT	Brazilian domestic input coefficients	20.9887	9.1546
	Brazilian import coefficients	3.3546	3.3321
	Total	0.4262	0.1917
Comparison 2: aggregate countries and compare with state trade in IRIO	Exports of Brazilian states	27.1520	25.2763
	Import coeffs of Brazilian states	34.1038	34.1253

Concerning the comparison 1, note that the IRIOT is based on (the Brazilian IO table which itself is based on) the official supply and use table (SUT) for 2008. The WIOT is based on the Brazilian SUT that was estimated in the WIOD project using the so-called SUT-RAS method (Timmer *et al.* 2012). Another inconsistency results from the fact that we were unable to obtain sectoral classification that was perfectly common for both datasets, which affects the services subsectors in particular. The detailed results show that the largest (absolute percentage) errors are found in sectors 21 (Transport), 22 (Post and Telecommunications; Other Business Activities), and 28 (Other Community, Social and Personal Services; Private Households with Employed Persons). These are precisely the sectors with classification problems. If we aggregate these three sectors (and thus remove the classification problems) and run the calculations at the 26-sector level, the WAPE for the domestic input coefficients drops considerably. In addition, it should be mentioned that the errors show both pluses and minuses. In our calculations (of the export/import of value added of Brazilian states) in the next section, the errors will cancel each other out to—at least—some extent. Therefore we have chosen to stick to the 28-sector classification.

In comparison 2, the WAPEs both for the vectors of state exports and for the vectors of state import coefficients are considerably larger than those in the first comparison. Note that the sectoral classification is not a main source of inconsistency in this case, as the WAPEs are not very different at the 26-sector level. Comparing the IRIOT aggregated at the national level and the Brazilian IO table from the WIOD project (which is compatible with the WIOT), we notice that the values of intermediate imports and exports by industry are different in each of the datasets, even though the totals are very close (total intermediate imports are approximately 6% larger in the WIOD dataset; total exports are almost identical). In the case of intermediate imports, it is due to the different procedures in the breakdown of the use table into domestic and imported origin. The Brazilian IO table that is basis for the IRIOT adopts the standard assumption of import proportionality, where the same, fixed percentage of total use of a product is assumed to be imported, irrespective of its purchaser. The WIOD project however, has developed an improved estimation method that does not rely on this standard import proportionality assumption[8]. Also the WIOD project's treatment for international margins responds for the differences in sectoral imports between the datasets. Valuation of the exports responds for the differences in their values by industry. In the IRIOT, departing from the values in purchasers' prices of the official SUT for 2008, trade and transport margins are subtracted from the exporting sectors and then are allocated to the respective margins sector. This valuation treatment, however, is not applied in the Brazilian input-output table from WIOD project. From this results that the values for national exports of sectors that are mainly producers of goods are larger according to the WIOT dataset than in the IRIOT, being the opposite valid for the services sectors (with the exception of sector 28, Other Community, Social and Personal Services; Private Households with Employed Persons). Given the valuation issue, the largest difference in the values of national exports between the datasets is found in sector 9 (Wholesale and retail trade).

Note that comparison 2 indicates that in our approach the international trade structure of Brazil (which is based on the WIOT) is quite different from that in the IRIOT. Since the WIOD project has developed an enhanced approach for global trade, we believe it is appropriate to base our estimates of Brazilian international trade on the WIOT. The WAPEs in comparison 2 indicate that improvements are being introduced in relation to the IRIOT.

Besides these differences in sectoral values of exports and imports for Brazil as whole, there is a source of inconsistency at the state level. Note that our approach disregards the vectors of total imports and exports by industry from the IRIOT (i.e. $\mathbf{m}^E$,

[8] The WIOD project's estimation method relies on a classification of detailed products in the International Trade Statistics, which enables the allocation of imports across end-use categories (i.e. 'intermediate consumption', 'final consumption', or 'gross fixed capital formation'). Then within each end-use category, the allocation was based on the proportionality assumption (Dietzenbacher et al., 2013).

$\mathbf{m}^W$, $\mathbf{e}^E$, and $\mathbf{e}^W$). The vectors of import and export shares are obtained from information that does not belong to the Brazilian IRIOT: for each region and for each product, how much is imported from / exported for each country. Such detailed data on foreign trade comes from AliceWeb[9] and it is not completely consistent with the figures both in the IRIOT and in the WIOT. Aggregating the AliceWeb's data over country of origin / destination does not result in the IRIOT's vectors of total imports and exports by state and by industry, as well aggregating the AliceWeb's data over state of destination / origin does not result in the WIOT's total figures of trade with Brazil (by country). Therefore discrepancies in the estimates for the vectors of state exports and for the vectors of state imports coefficients are expected.

In our approach, we have chosen not to take one of the datasets (say the WIOT) as a starting point and adapt the other dataset (i.e. the IRIOT) accordingly. In that case, comparison 1 would yield no errors, but the errors in comparison 2 would reflect all the inconsistencies between WIOT and IRIOT. Such an approach would be appropriate if one of the datasets is better than the other. In the current situation, we believe that is not the case. Therefore we have chosen to use "the best of both worlds" and construct input coefficients for which both datasets are used. Aggregating our data leads to inconsistencies with both the WIOT and the IRIOT. The robustness check in this section shows that the average errors remain reasonably small. Also the reported errors are comparable in size, which indicates that we are sort of "half way" between choosing for WIOT and choosing for IRIOT.

4. Results

In this section, we present some results from the empirical application.[10] Focusing its central question, we intend to analyze the Brazilian states' trade in value added, among them and the rest of the world. In order to contextualize them, these results are compared to those obtained for the countries in the WIOT. Finally, the Brazilian states' trade in value added with China and USA – the top trade partners of Brazil in 2008 – is analyzed, better illustrating the detailed results that our approach can provide.

[9] AliceWeb is the name of the System Of Analysis of Foreign Trade Information, of the Bureau of Foreign Trade, of the Brazilian Ministry of Development, Industry, and Foreign Trade.
[10] For detailed results (by region, by country, and by industry), please contact the authors.

3.1. Brazilian states' trade in value added

The results for international trade in value added of the Brazilian states – aggregated by all source / destination industries and countries, so that we can focus attention on the states – are presented in Table 1.

The first data column shows the total value added generated in the state that is embodied in the final use of other countries. The following column indicates the ratio between the exports in value added and the total value added generated in the state. The third column shows the total value added generated in other countries that is embodied in the final use of the state. The following column indicates the ratio between the imports in value added and the total value added generated in the state. The fifth column shows the surplus (exports less imports) of trade in value added of the state. The last column indicates the ratio between the surplus in value added and the total value added generated in the state.

Mato Grosso, for example, exported US$ 5471 million of value added in 2008 – that is, US$ 5471 million in value added generated in this state was embodied in the final use of other countries. On the other hand, imports of value added of Mato Grosso corresponded to US$ 1568 million – in other words, final use of Mato Grosso embodied US$ 1568 million in value added generated in other countries. So, in 2008, Mato Grosso presented a positive surplus of international trade in value added of US$ 3903 million, amounting in 15.5% of total value added generated in the state in that year.

Regarding the surplus of international trade in value added, we can observe that, in 2008, Amazonas presented the largest shortage, as its imports of value added greatly surpassed its exports. In contrast, Minas Gerais presented the largest positive surplus among all the Brazilian states, but it corresponded to only 4.1% of the total value added generated in the state in 2008. Relatively to that ratio, Pará stands out: its surplus of international trade amounted in 17.2% of total value added generated in the state in 2008. Also Mato Grosso stands out presenting net exports with a surplus larger than 10% of the states' value added (15.5% in fact).

At national level, as can be seem in the bottom of Table 1, turns out that the exports of value added (US$ 171056 million) were very close to the imports of value added (US$ 170859 million), resulting in a positive but small surplus of international trade in value added.

The bottom two rows of Table 1 disclose an important point regarding the Brazilian states' trade in value added. Both the exports and imports ratios to value added present higher weighted average across the Brazilian states (where the weights are the total value added generated in each state) than simple average. This fact indicates that states with higher value added export a larger share of their value added. In the same fashion, states with higher value added import products and services that amounts in a larger share of their value added.

Table 1. International Trade in Value Added, Brazilian States, 2008 – Million US$

	Exports	% Value Added	Imports	% Value Added	Surplus	% Value Added
Acre	137	4.1%	230	6.8%	-92	-2.8%
Amapá	182	5.3%	312	9.2%	-130	-3.8%
Amazonas	1,607	7.8%	6,205	30.0%	-4,599	-22.3%
Pará	7,964	27.2%	2,934	10.0%	5,029	17.2%
Rondônia	634	7.4%	773	9.0%	-139	-1.6%
Roraima	60	2.4%	142	5.8%	-83	-3.4%
Tocantins	422	6.6%	549	8.6%	-127	-2.0%
Alagoas	769	8.1%	734	7.8%	34	0.4%
Bahia	6,736	11.8%	7,737	13.6%	-1,002	-1.8%
Ceará	1,522	5.4%	2,870	10.1%	-1,348	-4.8%
Maranhão	2,322	12.2%	2,802	14.7%	-480	-2.5%
Paraíba	352	2.8%	1,365	11.0%	-1,013	-8.2%
Pernambuco	1,495	4.6%	4,144	12.7%	-2,650	-8.1%
Piauí	303	3.8%	864	10.8%	-561	-7.0%
Sergipe	627	6.6%	787	8.3%	-159	-1.7%
Rio Grande do Norte	720	6.0%	1,101	9.2%	-381	-3.2%
Distrito Federal	868	1.5%	4,391	7.8%	-3,523	-6.3%
Goiás	3,358	9.5%	4,067	11.5%	-709	-2.0%
Mato Grosso	5,471	21.7%	1,568	6.2%	3,903	15.5%
Mato Grosso do Sul	1,702	11.3%	1,839	12.2%	-136	-0.9%
Espírito Santo	6,391	22.0%	5,229	18.0%	1,161	4.0%
Minas Gerais	19,149	14.6%	13,805	10.5%	5,343	4.1%
Rio de Janeiro	19,835	12.6%	17,352	11.0%	2,482	1.6%
São Paulo	57,476	12.7%	57,765	12.7%	-289	-0.1%
Paraná	10,952	13.1%	11,324	13.5%	-372	-0.4%
Santa Catarina	6,699	11.6%	7,672	13.2%	-973	-1.7%
Rio Grande do Sul	13,304	14.2%	12,293	13.1%	1,011	1.1%
Total	171,056		170,859		197	
Average weighted		12.2%		12.2%		0.0%
Average unweighted		9.9%		11.4%		-1.5%

Concerning the Brazilian macroregions, Table 1 shows that trade in value added is highly concentrated in the states of Southeast and South. In 2008, these states responded for 78% of exported value added and for 73% of imported value added in Brazil. São Paulo alone accounted for 34% of both exported and imported value added in the country.

Table 1b. Gross Exports and Imports, International Trade in Value Added, Brazilian States, 2008 – Million US$

	Exports		Imports		Surplus	
	Gross value	Value added	Gross value	Value added	Gross value	Value added
Acre	39	137	224	230	-185	-92
Amapá	213	182	262	312	-50	-130
Amazonas	1.480	1.607	7.904	6.205	-6.424	-4.599
Pará	10.725	7.964	3.439	2.934	7.286	5.029
Rondônia	647	634	727	773	-80	-139
Roraima	30	60	125	142	-95	-83
Tocantins	330	422	555	549	-225	-127
Alagoas	979	769	754	734	226	34
Bahia	9.587	6.736	9.900	7.737	-313	-1.002
Ceará	1.513	1.522	3.158	2.870	-1.645	-1.348
Maranhão	2.654	2.322	3.574	2.802	-920	-480
Paraíba	311	352	1.254	1.365	-943	-1.013
Pernambuco	1.264	1.495	4.372	4.144	-3.108	-2.650
Piauí	177	303	750	864	-572	-561
Sergipe	173	627	830	787	-657	-159
Rio Grande do Norte	464	720	1.064	1.101	-600	-381
Distrito Federal	636	868	3.819	4.391	-3.183	-3.523
Goiás	4.386	3.358	4.675	4.067	-289	-709
Mato Grosso	8.071	5.471	2.109	1.568	5.962	3.903
Mato Grosso do Sul	2.201	1.702	3.271	1.839	-1.070	-136
Espírito Santo	10.192	6.391	4.011	5.229	6.180	1.161
Minas Gerais	26.109	19.149	17.844	13.805	8.265	5.343
Rio de Janeiro	20.991	19.835	21.917	17.352	-926	2.482
São Paulo	78.661	57.476	83.644	57.765	-4.982	-289
Paraná	16.119	10.952	16.090	11.324	29	-372
Santa Catarina	8.208	6.699	9.580	7.672	-1.372	-973
Rio Grande do Sul	19.766	13.304	16.932	12.293	2.834	1.011
Total	225.926	171.056	222.784	170.859	3.142	197

How do the results for Brazil and its states relate to other countries? In order to contextualize them, we present Table 2, which is equivalent to Table 1 but was computed for selected countries – those with the highest values of trade in value added and other developing countries – and for the WIOT as a whole.

Table 2. Trade in Value Added, Selected Countries and Total, 2008 – Million US$

	Exports	% Value Added	Imports	% Value Added	Surplus	% Value Added
Brazil	171,056	12.2%	170,859	12.2%	197	0.0%
Canada	372,396	26.5%	329,022	23.4%	43,373	3.1%
China	1,525,398	34.6%	896,192	20.3%	629,206	14.3%
Germany	1,049,941	32.1%	758,659	23.2%	291,281	8.9%
France	449,716	17.6%	501,144	19.6%	-51,428	-2.0%
UK	540,362	22.1%	551,867	22.5%	-11,505	-0.5%
Indonesia	117,124	24.1%	100,465	20.7%	16,659	3.4%
India	183,084	15.4%	217,587	18.3%	-34,504	-2.9%
Japan	676,130	14.2%	596,165	12.5%	79,966	1.7%
Korea	246,761	29.7%	247,194	29.7%	-433	-0.1%
Mexico	190,185	18.4%	188,421	18.2%	1,764	0.2%
Netherlands	290,840	37.5%	222,624	28.7%	68,216	8.8%
Russia	355,670	26.0%	262,694	19.2%	92,976	6.8%
USA	1,302,756	9.0%	1,891,039	13.1%	-588,283	-4.1%
RoW	2,172,022	27.2%	2,564,064	32.1%	-392,042	-4.9%
Total	11,958,645		11,958,645		0	
Average weighted		20.7%		20.7%		0.0%
Average unweighted		28.3%		29.3%		-1.0%

From Table 2, we can observe that the average country is much more involved in the global value chain than Brazil is. The second to last row indicates that the average country presents approximately twice as much trade in value added, as % of country's value added. Still paying attention to the bottom rows of Table 2, it is interesting that both the exports and imports ratios to value added present higher simple average across the countries than weighted average – the opposite to what was observed for the Brazilian states. In other words, we can point out that in average, countries with little value added (i.e. small countries) export a larger share of their value added than countries with higher value added (i.e. large countries). The same conclusion holds for the imports. The most illustrative country to this point is, in Table 2, the Netherlands. Among others, we can point out as distinguished small, open countries in the WIOT, omitted in the Table: Luxembourg (which exports of value added amounted in 61.4% of total value added of the country) and Bulgaria (which imports of value added amount in 57.5% of its total value added).

Observing the results for each country, we can point out that the surplus of trade in value added close to zero presented by Brazil is a special case among the countries in the WIOT. Besides Brazil, only UK, Korea, Mexico, and Slovakia (omitted in Table 2) presented, in 2008, a ratio between surplus of trade in value added and total value added in the interval [-0.01, 0.01].

China stands out in Table 2, presenting the highest positive surplus of trade in value added among all the countries in the WIOT. In 2008, that Chinese surplus amounted in 14.3% of the total value added in the country. On the other hand, there is the case of USA – in 2008, according to Table 2, the country's large shortage corresponded to 4.1% of its total value added. The relationship of the Brazilian states with these two countries will be further investigated below.

3.2. The Brazilian value chain

We have observed that Brazilian trade in value added is somewhat limited. So, it is interesting to more carefully analyze the trade of Brazilian states. Do they trade? And, if yes, with whom? Table 3 aims to add to this question, showing the composition of trade in value added of the Brazilian states, as in 2008.

For example, according to Table 3, in 2008, 54% of Paraná's value added was generated for its own final use (domestic share), 32% for rest of Brazil's final use (regional exports' share), and – found by difference – 13% for other countries' final use. Moreover, Paraná's final use led to purchases in value added from other Brazilian states amounted in 27% of its own total value added (regional imports' share). As a result, Paraná's regional trade presented a positive surplus corresponding to 6% of its value added

Table 3. Domestic and Regional Shares in Total Trade in Value Added, Brazilian States, 2008

	Domestic	Regional exports	Regional imports	Regional Surplus	Intnl Surplus
Acre	73%	22.5%	34.4%	-12%	-3%
Amapá	82%	12.6%	42.4%	-30%	-4%
Amazonas	41%	51.0%	30.7%	20%	-22%
Pará	57%	16.2%	44.2%	-28%	17%
Rondônia	68%	25.1%	42.4%	-17%	-2%
Roraima	84%	13.9%	24.0%	-10%	-3%
Tocantins	64%	29.6%	37.9%	-8%	-2%
Alagoas	69%	22.5%	35.2%	-13%	0%
Bahia	68%	19.9%	40.4%	-21%	-2%
Ceará	68%	26.8%	30.8%	-4%	-5%
Maranhão	62%	25.5%	38.3%	-13%	-3%
Paraíba	80%	16.8%	47.2%	-30%	-8%
Pernambuco	76%	19.8%	39.0%	-19%	-8%
Piauí	81%	15.3%	60.2%	-45%	-7%
Sergipe	63%	30.8%	36.8%	-6%	-2%
Rio Grande do Norte	72%	22.0%	39.6%	-18%	-3%
Distrito Federal	88%	10.1%	31.4%	-21%	-6%
Goiás	63%	27.4%	34.5%	-7%	-2%
Mato Grosso	44%	34.2%	23.8%	10%	15%
Mato Grosso do Sul	61%	28.1%	36.4%	-8%	-1%
Espírito Santo	48%	30.5%	34.5%	-4%	4%
Minas Gerais	61%	24.3%	32.6%	-8%	4%
Rio de Janeiro	64%	23.4%	30.8%	-7%	2%
São Paulo	57%	30.5%	14.1%	16%	0%
Paraná	54%	32.4%	26.7%	6%	0%
Santa Catarina	57%	31.4%	30.2%	1%	-2%
Rio Grande do Sul	61%	25.1%	27.8%	-3%	1%

Regarding the domestic share of the responsibility for value added generated in the Brazilian states, a first note is that in general it is somewhat smaller than for countries in the WIOT (81.3% in average). That share is notably small in Amazonas: only 41% of its value added was due to its own final use. In the other hand, Distrito Federal's final use was greatly responsible for its own value added (88%).

The note that the domestic shares presented in Table 3 are smaller than those for countries is fairly expected, since states might trade with each other inside a country. We can point out that this is the case for Brazilian states: the average regional exports' share was 24.7%, in 2008. Amazonas is especially outstanding, as approximately half of its value added was due to the final use of other states in the country. Besides, the average regional imports' share was 35.1% – reaching the level of 60% in Piauí.

Therefore, it is possible to answer the question motivating Table 3: yes, Brazilian states do trade, and mainly with each other.

The two final columns in Table 3 present the surplus of each state from regional and international trade in value added. From them, it is remarkable that 16 out of 27 states had ‘two shortages’ in 2008, and Mato Grosso was the only state with ‘two surpluses’.

Concerning the regional trade in value added, since by definition there is balance in the national level, we observe that the surplus of 16% for São Paulo ‘does the job’. The total of regional surpluses in 2008 was US$ 86667 million, of which São Paulo was responsible for 85.8%. This fact indicates that there were net imports by other states from São Paulo, what can be observed in Table 4, which is analogous to Table 1 but considers Brazilian states’ direct and indirect trade in value added with São Paulo.

From Table 4, in 2008 only Amazonas was not a net importer of value added from São Paulo. However, even for Amazonas, São Paulo was the main source (other than the own state) of value added. The relevance of São Paulo for trade in Brazil is reinforced by the fact that it was also the main importer of value added for 25 out of 27 states – the exceptions are Pará and Distrito Federal, which largest share of exports of value added went to USA and Minas Gerais, respectively.

Table 4. Trade in Value Added with São Paulo, Brazilian States, 2008 – Million US$

	Exports	% Value Added	Imports	% Value Added	Surplus	% Value Added
Acre	151	4.5%	437	13.0%	-285	-8.5%
Amapá	80	2.4%	585	17.2%	-505	-14.9%
Amazonas	3,722	18.0%	2,928	14.2%	794	3.8%
Pará	1,049	3.6%	4,941	16.9%	-3,892	-13.3%
Rondônia	506	5.9%	1,528	17.8%	-1,022	-11.9%
Roraima	73	3.0%	233	9.5%	-160	-6.5%
Tocantins	333	5.2%	952	14.9%	-619	-9.7%
Alagoas	454	4.8%	1,228	13.0%	-774	-8.2%
Bahia	3,720	6.5%	10,235	17.9%	-6,516	-11.4%
Ceará	1,583	5.6%	2,951	10.4%	-1,368	-4.8%
Maranhão	1,084	5.7%	2,633	13.8%	-1,549	-8.1%
Paraíba	458	3.7%	2,066	16.7%	-1,609	-13.0%
Pernambuco	1,418	4.4%	4,860	14.9%	-3,442	-10.6%
Piauí	241	3.0%	1,720	21.5%	-1,479	-18.5%
Sergipe	657	6.9%	1,229	12.9%	-572	-6.0%
Rio Grande do Norte	732	6.1%	1,661	13.8%	-929	-7.7%
Distrito Federal	594	1.1%	7,795	13.9%	-7,201	-12.8%
Goiás	2,580	7.3%	5,541	15.7%	-2,960	-8.4%
Mato Grosso	1,869	7.4%	2,626	10.4%	-758	-3.0%
Mato Grosso do Sul	1,328	8.8%	2,461	16.4%	-1,133	-7.5%
Espírito Santo	2,561	8.8%	3,989	13.7%	-1,428	-4.9%
Minas Gerais	10,333	7.9%	21,689	16.6%	-11,356	-8.7%
Rio de Janeiro	10,080	6.4%	24,136	15.3%	-14,055	-8.9%
Paraná	7,578	9.0%	10,312	12.3%	-2,734	-3.3%
Santa Catarina	4,409	7.6%	7,736	13.4%	-3,327	-5.7%
Rio Grande do Sul	6,460	6.9%	11,966	12.8%	-5,506	-5.9%
Total	64,051		138,435		-74,384	
Average weighted		6.8%		14.6%		-7.9%
Average unweighted		6.2%		14.6%		-8.4%

By calculating the ratio between the value added embodied in final use of a state and the value added generated in a state, we obtain what can be called a ‘value added footprint’. The results are presented in Table 5.

Table 5. 'Value Added Footprints', Brazilian States, 2008

	Value added footprints
Acre	1.15
Amapá	1.34
Amazonas	1.02
Pará	1.11
Rondônia	1.19
Roraima	1.13
Tocantins	1.10
Alagoas	1.12
Bahia	1.22
Ceará	1.09
Maranhão	1.15
Paraíba	1.39
Pernambuco	1.27
Piauí	1.52
Sergipe	1.08
Rio Grande do Norte	1.21
Distrito Federal	1.28
Goiás	1.09
Mato Grosso	0.74
Mato Grosso do Sul	1.09
Espírito Santo	1.00
Minas Gerais	1.04
Rio de Janeiro	1.06
São Paulo	0.84
Paraná	0.95
Santa Catarina	1.00
Rio Grande do Sul	1.02

According to Table 5, only Mato Grosso, São Paulo, and Paraná had value added footprints lower than 1 in 2008, i.e., they generated greater amounts of value added than those required by their final use (from them, other states and countries, directly and indirectly). This indicates that those states have important roles in the Brazilian trade as net suppliers of value added.

3.3. Trade with China and USA

In order to better illustrate the detail level of our empirical application, we analyze below the trade between the Brazilian states and the two top trade partners of Brazil in 2008, China and USA. Some worldwide results for these countries were presented in the previous subsection.

According to Table 2, in 2008 China presented an outstanding positive surplus of trade in value added worldwide. In order to investigate if this is also the case for the Brazilian states' trade relations with the country, Table 6 presents results referring to direct and indirect trade in value added with China.

Table 6. Trade in Value Added with China, Brazilian States, 2008 – Million US$

	Exports	% Value Added	Imports	% Value Added	Surplus	% Value Added
Acre	12	0.3%	21	0.6%	-9	-0.3%
Amapá	15	0.4%	32	0.9%	-18	-0.5%
Amazonas	88	0.4%	1,920	9.3%	-1,832	-8.9%
Pará	1,200	4.1%	275	0.9%	924	3.2%
Rondônia	39	0.5%	88	1.0%	-48	-0.6%
Roraima	3	0.1%	12	0.5%	-9	-0.4%
Tocantins	70	1.1%	56	0.9%	14	0.2%
Alagoas	29	0.3%	75	0.8%	-45	-0.5%
Bahia	465	0.8%	805	1.4%	-341	-0.6%
Ceará	75	0.3%	291	1.0%	-216	-0.8%
Maranhão	378	2.0%	180	0.9%	199	1.0%
Paraíba	17	0.1%	193	1.6%	-177	-1.4%
Pernambuco	75	0.2%	361	1.1%	-285	-0.9%
Piauí	33	0.4%	84	1.0%	-51	-0.6%
Sergipe	36	0.4%	74	0.8%	-38	-0.4%
Rio Grande do Norte	32	0.3%	101	0.8%	-69	-0.6%
Distrito Federal	48	0.1%	320	0.6%	-273	-0.5%
Goiás	467	1.3%	349	1.0%	118	0.3%
Mato Grosso	893	3.5%	141	0.6%	752	3.0%
Mato Grosso do Sul	216	1.4%	185	1.2%	31	0.2%
Espírito Santo	613	2.1%	945	3.3%	-332	-1.1%
Minas Gerais	1,266	1.0%	1,415	1.1%	-149	-0.1%
Rio de Janeiro	3,784	2.4%	1,514	1.0%	2,269	1.4%
São Paulo	3,237	0.7%	6,695	1.5%	-3,458	-0.8%
Paraná	1,101	1.3%	1,345	1.6%	-244	-0.3%
Santa Catarina	353	0.6%	1,121	1.9%	-768	-1.3%
Rio Grande do Sul	1,374	1.5%	1,070	1.1%	304	0.3%
Total	15,917		19,667		-3,750	
Average weighted		1.1%		1.4%		-0.3%
Average unweighted		1.0%		1.4%		-0.4%

At national level, there was a shortage of trade in value added with China, i.e., Brazil was indeed a net importer of Chinese value added in 2008 as its imports surpassed its exports in US$ 3750 million.

Concerning the Brazilian states' trade with China, Table 6 shows that only 8 out of 27 states had a positive surplus of trade in value added. Pará and notably Rio de Janeiro presented the greatest surpluses. For both states, their industry 'Mining and Quarrying' was largely responsible for the value added embodied in Chinese final use. São Paulo was also an important exporter of value added to China, but in the case of this state such value added was generated in several industries, notably those concerning services (47.2%) and manufacturing (45.4%). São Paulo, however, presented a shortage in its trade in value added with China. The largest share (61%) of Chinese value added embodied in São Paulo's final use was generated by manufacturing industries, remarkably the industry 'Electrical and Optical Equipment', which generated US$ 1290 million in value added directly and indirectly due to São Paulo's final use in 2008.

In contrast with the Chinese case, Table 2 indicates that in 2008 USA had a large shortage in its trade in value added. The results obtained for the Brazilian states' trade in value added with USA are summarized in Table 7 below.

Table 7. Trade in Value Added with USA, Brazilian States, 2008 – Million US$

	Exports	% Value Added	Imports	% Value Added	Surplus	% Value Added
Acre	11	0.3%	26	0.8%	-15	-0.4%
Amapá	39	1.1%	36	1.1%	3	0.1%
Amazonas	245	1.2%	612	3.0%	-367	-1.8%
Pará	1,712	5.8%	401	1.4%	1,311	4.5%
Rondônia	52	0.6%	87	1.0%	-35	-0.4%
Roraima	6	0.2%	16	0.7%	-10	-0.4%
Tocantins	30	0.5%	61	1.0%	-31	-0.5%
Alagoas	102	1.1%	89	0.9%	13	0.1%
Bahia	1,587	2.8%	828	1.5%	759	1.3%
Ceará	267	0.9%	304	1.1%	-37	-0.1%
Maranhão	368	1.9%	340	1.8%	29	0.2%
Paraíba	83	0.7%	150	1.2%	-67	-0.5%
Pernambuco	225	0.7%	486	1.5%	-260	-0.8%
Piauí	32	0.4%	99	1.2%	-67	-0.8%
Sergipe	106	1.1%	98	1.0%	8	0.1%
Rio Grande do Norte	143	1.2%	137	1.1%	6	0.0%
Distrito Federal	91	0.2%	512	0.9%	-421	-0.7%
Goiás	250	0.7%	477	1.3%	-227	-0.6%
Mato Grosso	344	1.4%	191	0.8%	153	0.6%
Mato Grosso do Sul	173	1.2%	188	1.2%	-14	-0.1%
Espírito Santo	1,600	5.5%	690	2.4%	910	3.1%
Minas Gerais	3,300	2.5%	1,626	1.2%	1,674	1.3%
Rio de Janeiro	2,645	1.7%	2,333	1.5%	312	0.2%
São Paulo	9,847	2.2%	7,826	1.7%	2,021	0.4%
Paraná	1,048	1.3%	1,197	1.4%	-149	-0.2%
Santa Catarina	941	1.6%	813	1.4%	129	0.2%
Rio Grande do Sul	2,127	2.3%	1,323	1.4%	804	0.9%
Total	27,376		20,945		6,432	
Average weighted		2.0%		1.5%		0.5%
Average unweighted		1.5%		1.3%		0.2%

At national level, in 2008 there were net exports of value added from Brazil to USA, with a surplus of US$ 6423 million. In other words, USA's final use caused the generation of higher levels of value added in the industries of Brazil than the other way around.

Regarding the states' trade in value added with USA, as Table 7 shows, 14 out 27 Brazilian states presented a positive surplus. The industries 'Mining and Quarrying' and 'Basic Metals and Fabricated Metal' were crucial to the surpluses in value added presented by Pará and Minas Gerais. The largest surplus, however, corresponded to São Paulo. The São Paulo's value added embodied in the USA's final use was generated mainly in the manufacturing industries (49.8%) and services (36.8%, with special weight of the industry ‘Post and Telecommunications; Other Business Activities’), but the industry ‘Mining and Quarrying’ was also responsible for a sizable share (9.7%). Despite this surplus, São Paulo was also, by far, the state which final use embodied the greatest amount of USA’s value added. Those imports of value added by São Paulo were generated mainly in manufacturing industries (50.1%, with great contribution of the industries ‘Electrical and Optical Equipment’ and ‘Chemicals and Chemical Products’) and in services (44.4%, notably by the industry ‘Post and Telecommunications; Other Business Activities’).

5. Concluding Remarks

The fact that many of today's products and services are no longer produced within a single country raises many relevant questions. Among them, what are the effects of fragmentation of production on the distribution of income across countries, regions within countries and industries? The present paper aimed to contribute to this discussion, analyzing the generation of value added in the Brazilian states in the context of global value chains. Here, we focused on the value added variable in the spirit of the OECD-WTO 'Made in the World' initiative, which proposes 'trade in value added' as a better approach for the measurement for international trade.

Since international fragmentation of production processes leads to an interdependent structure which has to be accounted for, the input-output methodology is especially suitable. In our analysis, we combined a world input-output table from the WIOD project with an inter-regional input-output table. From this, in the empirical application, we obtained a model covering the interdependence of 27 Brazilian states and 40 other countries (with the rest of the world as a country) in 2008, with the economic structures arranged in 28 industries. An important derivation of this new theoretical development is that this methodology can also be applied to the analysis of the integration of regions in other countries to the GVC.

In our results, we observed that the average country trades approximately twice as much in value added (as a share of country's value added) than Brazil. Therefore, the participation of Brazil in the global value chain is somewhat limited. Another interesting point is that, in 2008, the trade in value added for Brazil as a whole presented a surplus close to zero, being its exports nearly equal to its imports. There is, however, heterogeneity among the Brazilian states: some states (e.g. Amazonas) present large shortages in international trade in value added, while others (e.g. Pará) present positive surpluses. Besides this, in absolute terms, states in Southeast and South dominate the Brazilian international trade in value added.

The Brazilian value chain was, then, further analyzed for the year 2008. We indicated that 16 out of 27 states presented shortages for both regional and international trade, Mato Grosso being the only state with two positive surpluses. A relevant remark is that

São Paulo’s surplus in regional trade does the job of balancing the transactions among Brazilian states: almost all states (Amazonas as the only exception) are net importers of value added from São Paulo.

Finally, the Brazilian states’ trade in value added with China and USA, the top trade partners of the country in 2008, were analyzed displaying the results’ detail level that our model can provide. It was verified that in national terms there was a shortage in trade in value added with China and a surplus with USA. However, we noticed many heterogeneities among states, both in terms of trade volume and of industries that account for the generation of value added in the states and in their trade partners.

Many other questions that were raised with the global value chains can be discussed with the model that was developed in the present paper and its underlying database. It allows for addressing issues related to other socio-economic aspects (such as employment by skill type) as well as environmental aspects (such as energy use, various emissions to air, or the use of water). Therefore, future works can greatly benefit from the model to analyze questions concerning the Brazilian states’ trade relations in a global value chain.

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Appendix A

List of the 28 industries in the model:

Industry	Number
Agriculture, Hunting, Forestry and Fishing	1
Mining and Quarrying	2
Food, Beverages and Tobacco	3
Textiles and Textile Products	4
Leather, Leather and Footwear	5
Wood and Products of Wood and Cork	6
Pulp, Paper, Paper , Printing and Publishing	7
Coke, Refined Petroleum and Nuclear Fuel	8
Chemicals and Chemical Products	9
Rubber and Plastics	10
Other Non-Metallic Mineral	11
Basic Metals and Fabricated Metal	12
Machinery, Nec	13
Electrical and Optical Equipment	14
Transport Equipment	15
Manufacturing, Nec; Recycling	16
Electricity, Gas and Water Supply	17
Construction	18
Wholesale and retail trade	19
Hotels and Restaurants	20
Transport	21
Post and Telecommunications; Other Business Activities	22
Financial Intermediation	23
Real Estate Activities	24
Public Admin and Defence; Compulsory Social Security	25
Education	26
Health and Social Work	27
Other Community, Social and Personal Services; Private Households with Employed Persons	28

List of the 67 regions in the model:

Region	Number	Region	Number	Region	Number
Acre	1	Australia	28	Malta	55
Amapá	2	Austria	29	Netherlands	56
Amazonas	3	Belgium	30	Poland	57
Pará	4	Bulgaria	31	Portugal	58
Rondonia	5	Canada	32	Romania	59
Roraima	6	China	33	Russia	60
Tocantins	7	Cyprus	34	Slovak Republic	61
Alagoas	8	Czech Republic	35	Slovenia	62
Bahia	9	Germany	36	Sweden	63
Ceará	10	Denmark	37	Turkey	64
Maranhão	11	Spain	38	Taiwan	65
Paraiba	12	Estonia	39	USA	66
Pernambuco	13	Finland	40	RoW	67
Piauí	14	France	41		
Sergipe	15	United Kingdom	42		
Rio Grande do Norte	16	Greece	43		
Distrito Federal	17	Hungary	44		
Goiás	18	Indonesia	45		
Mato Grosso	19	India	46		
Mato Grosso do Sul	20	Ireland	47		
Espírito Santo	21	Italy	48		
Minas Gerais	22	Japan	49		
Rio de Janeiro	23	Korea	50		
São Paulo	24	Lithuania	51		
Paraná	25	Luxembourg	52		
Santa Catarina	26	Latvia	53		
Rio Grande do Sul	27	Mexico	54		

Appendix B

The combination of the WIOT and the Brazilian IRIOT yields a final demand vector and the input coefficients matrix in (7). We may thus calculate an output vector

$$\begin{pmatrix} \tilde{\mathbf{x}}^R \\ \tilde{\mathbf{x}}^S \\ \tilde{\mathbf{x}}^E \\ \tilde{\mathbf{x}}^W \end{pmatrix} = \left(\mathbf{I} - \begin{bmatrix} \mathbf{A}^{RR} & \mathbf{A}^{RS} & \tilde{\mathbf{A}}^{RE} & \tilde{\mathbf{A}}^{RW} \\ \mathbf{A}^{SR} & \mathbf{A}^{SS} & \tilde{\mathbf{A}}^{SE} & \tilde{\mathbf{A}}^{SW} \\ \tilde{\mathbf{A}}^{ER} & \tilde{\mathbf{A}}^{ES} & \mathbf{A}^{EE} & \mathbf{A}^{EW} \\ \tilde{\mathbf{A}}^{WR} & \tilde{\mathbf{A}}^{WS} & \mathbf{A}^{WE} & \mathbf{A}^{WW} \end{bmatrix} \right)^{-1} \left\{ \begin{pmatrix} \mathbf{c}^{RR} \\ \mathbf{c}^{SR} \\ \hat{\boldsymbol{\sigma}}^{ER}\mathbf{c}^{TR} \\ \hat{\boldsymbol{\sigma}}^{WR}\mathbf{c}^{TR} \end{pmatrix} + \begin{pmatrix} \mathbf{c}^{SR} \\ \mathbf{c}^{SS} \\ \hat{\boldsymbol{\sigma}}^{ES}\mathbf{c}^{TS} \\ \hat{\boldsymbol{\sigma}}^{WS}\mathbf{c}^{TS} \end{pmatrix} + \begin{pmatrix} \hat{\boldsymbol{\lambda}}^{RE}\mathbf{c}^{RT} \\ \hat{\boldsymbol{\lambda}}^{SE}\mathbf{c}^{ST} \\ \mathbf{c}^{EE} \\ \mathbf{c}^{WE} \end{pmatrix} + \begin{pmatrix} \hat{\boldsymbol{\lambda}}^{RW}\mathbf{c}^{RT} \\ \hat{\boldsymbol{\lambda}}^{SW}\mathbf{c}^{ST} \\ \mathbf{c}^{EW} \\ \mathbf{c}^{WW} \end{pmatrix} \right\}$$

The first comparison is based on an estimate $\tilde{\mathbf{A}}^{TT}$ of the Brazilian domestic input coefficients. The aggregated Brazilian intra-country deliveries are obtained as

$$\tilde{\mathbf{Z}}^{TT} = \mathbf{A}^{EE}\hat{\tilde{\mathbf{x}}}^{E} + \mathbf{A}^{WE}\hat{\tilde{\mathbf{x}}}^{E} + \mathbf{A}^{EW}\hat{\tilde{\mathbf{x}}}^{W} + \mathbf{A}^{WW}\hat{\tilde{\mathbf{x}}}^{W}$$

Defining $\tilde{\mathbf{x}}^T = \tilde{\mathbf{x}}^E + \tilde{\mathbf{x}}^W$, we have $\tilde{\mathbf{A}}^{TT} = \tilde{\mathbf{Z}}^{TT}\left(\hat{\tilde{\mathbf{x}}}^{T}\right)^{-1}$. The estimates $\tilde{\mathbf{A}}^{RT}$ and $\tilde{\mathbf{A}}^{ST}$ are obtained in a similar way. For example, $\tilde{\mathbf{Z}}^{RT} = \mathbf{A}^{RE}\hat{\tilde{\mathbf{x}}}^{E} + \mathbf{A}^{RW}\hat{\tilde{\mathbf{x}}}^{W}$ and $\tilde{\mathbf{A}}^{RT} = \tilde{\mathbf{Z}}^{RT}\left(\hat{\tilde{\mathbf{x}}}^{T}\right)^{-1}$.

The second comparison is based on estimates for the vectors of state exports (i.e. $\tilde{\mathbf{e}}^{ER}$, $\tilde{\mathbf{e}}^{ES}$, $\tilde{\mathbf{e}}^{WR}$, and $\tilde{\mathbf{e}}^{WS}$) and for the vectors of state import coefficients (i.e. estimates of $(\mathbf{m}^E)'(\hat{\mathbf{x}}^E)^{-1}$ and $(\mathbf{m}^W)'(\hat{\mathbf{x}}^W)^{-1}$). For example, for the state exports we have

$$\tilde{\mathbf{e}}^{ER} = \hat{\boldsymbol{\sigma}}^{ER}\mathbf{A}^{TR}\tilde{\mathbf{x}}^{R} + \hat{\boldsymbol{\sigma}}^{ER}\mathbf{c}^{TR}$$

The row vector with estimates for the state import coefficients (for example for region *E*) is obtained as

$$(\boldsymbol{\lambda}^{RE})'\mathbf{A}^{RE}(\hat{\boldsymbol{\mu}}^{E})^{-1} + (\boldsymbol{\lambda}^{SE})'\mathbf{A}^{SE}(\hat{\boldsymbol{\mu}}^{E})^{-1}$$