2019/2020

Topology of manifolds

Code: 100114
ECTS Credits: 6

Degree	Type	Year	Semester
2500149 Mathematics	OT	4	0

Contact

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Use of Languages

Principal working language: spanish (spa)

Some groups entirely in English: No

Some groups entirely in Catalan: No

Some groups entirely in Spanish: Yes

Teachers

Juan Luis Durán Batalla

Prerequisites

It is better to have succeded in the course "Diferential Calculus".

Objectives and Contextualisation

Ever since the concept of homeomorphism was clearly defined, the "ultimate" problem in topology has been to classify topological spaces "up to homeomorphism". That this was a hopeless undertaking was very soon apparent, the subspaces of the plane R2 being an obvious example. From this impossibility were born algebraic and differential topology, by a shift of emphasis which consisted in associating "invariant" objects to some types of spaces (objects are the same for two homeomorphic spaces). At first these objects were integers, but it was soon realized that much more information could be extracted from invariant algebraic structures such as groups and rings.

(Jean Dieudonné, A history of algebraic and differential topology 1900--1960)

If topology is concerend with the classification of shapes, in this course we will introduce some of the most basic algebraic tools used in this classification: de Rahm cohomology and the fundamental group. In particular de Rahm cohomology, a natural genralization of differential calculus, attaches to each manifold a series of finite dimensional vector spaces which encode various features of a manifold: its dimension, its orientability, higher orientability properties (spin structures etc.). The objective of this course is to construct these vector spaces and present some of the tools used to extract from them relevant information.

Competences

- Develop critical thinking and reasoning and know how to communicate it effectively, both in ones own languages and in a third language.
- Generate innovative and competitive proposals for research and professional activities.
- Students must be capable of communicating information, ideas, problems and solutions to both specialised and non-specialised audiences.

- Students must have and understand knowledge of an area of study built on the basis of general secondary education, and while it relies on some advanced textbooks it also includes some aspects coming from the forefront of its field of study.

Learning Outcomes

1. Develop critical thinking and reasoning and know how to communicate it effectively, both in ones own languages and in a third language.
2. Generate innovative and competitive proposals for research and professional activities.
3. Students must be capable of communicating information, ideas, problems and solutions to both specialised and non-specialised audiences.
4. Students must have and understand knowledge of an area of study built on the basis of general secondary education, and while it relies on some advanced textbooks it also includes some aspects coming from the forefront of its field of study.

Content

We will cover the following topics:

- Smooth manifolds.
- Smooth atlas, change of charts.
- Tangent and cotangent bundles
- Vector fields

- De Rahm complex
- Differential forms
- De Rahm complex
- Cohomology
- Euler characteristic
- Poincaré duality

As an application we will prove some of these results:

- Classification of closed surfaces
- Brouwers fixed point theorem
- Jordan-Brouwer separation theorem
- Topological invariance of dimension.

Methodology

The course consists of theory classes, where the theory is explained and the important proofs are detailed to the sutendts; problems sessions and seminars where one can both practice the concepts explained in the theory and to go more into the details of some of the concepts or examples.

Activities

Title	Hours	ECTS	Learning Outcomes
Type: Directed			
Problem Sessions	14	0.56	1, 2, 4, 3
Theory classes	30	1.2	1, 2, 4
Type: Supervised			

Seminars	6	0.24	1, 2, 4
Type: Autonomous			
Assimilations of theoretical results	45	1.8	2, 4, 3
Homework	15	0.6	1, 4, 3
Solving problems	30	1.2	4, 3

Assessment

The fianl grade of the course is obtained as follows:

0.4 Final exam + 0.4 Mid Term + 0.2 Homework.

The reevaluation exam replaces both the Mid Term exam and the Final exam. The homework can not be reevaluated.

A student that does not go to the Mid Term exam will be graded as "No Evaluable".

Assessment Activities

Title	Weighting	Hours	ECTS	Learning Outcomes
Final Exam	40%	3	0.12	1, 4
Homework	20%	1	0.04	1, 2, 4, 3
Mid Term	40	3	0.12	1, 4
Reevaluation	80%	3	0.12	1, 2, 4, 3

Bibliography

A full set of notes for the course written by the professor is provided during the course.

R. Bott and L.W. Tu, Differential forms in algebraic topology. Graduate Texts in Mathematics, 82. Springer-Verlag, New York-Berlin, 1982. xiv+331 pp.

A. Hatcher, Algebraic topology. Cambridge University Press, Cambridge, 2002. xii+544 pp. (http://www.math.cornell.edu/~hatcher/AT/ATpage.html).

I. Madsen and J. Tornehave, From calculus to cohomology. de Rham cohomology and characteristic classes. *Cambridge University Press, Cambridge,* 1997. viii+286 pp.

W.S. Massey, Algebraic Topology: An introduction. Graduate Texts in Mathematics, 56. Springer-Verlag, New York, 1977. xxi+261 pp.

V. Navarro and P. Pascual: Topologia Algebraica. Edicions UB, 34, Barcelona, 1999. 326pp.

I.M. Singer and J.A. Thorpe, Lecture notes on elementary topology and geometry. Undergraduate Texts in Mathematics. Springer-Verlag, New York-Heidelberg, 1976. viii+232 pp.

J. W. Vick, Homology theory. An introduction to algebraic topology. Second edition. Graduate Texts in Mathematics, 145. Springer-Verlag, New York, 1994. xiv+242 pp.

F.W. Warner, Foundations of differentiable manifolds and Lie groups. Graduate Texts in Mathematics, 94. Springer-Verlag, New York-Berlin, 1983. ix+272 pp.