Universitat Autònoma de Barcelona

2019/2020

Linear geometry

Code: 100095
ECTS Credits: 6

Degree	Type	Year	Semester
2500149 Mathematics	OB	2	1

Contact

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Use of Languages

Principal working language: catalan (cat)
Some groups entirely in English: No
Some groups entirely in Catalan: Yes
Some groups entirely in Spanish: No

Teachers

Jaume Aguadé Bover
Roberto Rubio Nuñez
David Marín Pérez

Prerequisites

The main prerequisites for this course are the 1st year courses "Linear Algebra" and "Foundations of Mathematics".

Objectives and Contextualisation

The main goal in this course is to present the fundamental ideas of this millennium-old subject called Geometry.

After a short discussion about "what is Geometry?", we analyze the axiomatic method contained in the work of Euclid and Hilbert and we discuss how it leads to different geometries.

Next, we introduce the "projective viewpoint" and we learn how natural it is to include in our space the "points at infinity". We continue with an introduction to projective and affine spaces and their transformations.

The last section is about quadrics and we will learn how to see them from four different (but equivalent) points of view: bilinear forms, quadratic forms, second degree polynomials and quadrics as geometric objects in projective or affine space.

Throughout the course, we will not limit our attention to the real field. The very important cases of finite fields and finite geometries will not be underestimated because in the digital era they play a crucial role in many applications of geometry.

Competences

- Assimilate the definition of new mathematical objects, relate them with other contents and deduce their properties.
- Identify the essential ideas of the demonstrations of certain basic theorems and know how to adapt them to obtain other results.
- Students must be capable of applying their knowledge to their work or vocation in a professional way and they should have building arguments and problem resolution skills within their area of study.
- Students must have and understand knowledge of an area of study built on the basis of general secondary education, and while it relies on some advanced textbooks it also includes some aspects coming from the forefront of its field of study.
- Use computer applications for statistical analysis, numeric and symbolic calculus, graphic display, optimisation or other purposes to experiment with Mathematics and solve problems.

Learning Outcomes

1. Classify conic and quadric sections and find their notable elements.
2. Classify planar and spatial isometrics, determining the type and characteristic elements.
3. Know how to resolve planar and spatial geometric problems.
4. Operate with points, vectors, distances and angles in relative and Euclidian spaces and with the corresponding systems of reference, subspaces and transformations
5. Students must be capable of applying their knowledge to their work or vocation in a professional way and they should have building arguments and problem resolution skills within their area of study.
6. Students must have and understand knowledge of an area of study built on the basis of general secondary education, and while it relies on some advanced textbooks it also includes some aspects coming from the forefront of its field of study.

Content

Foundations of geometry
1. What is geometry?
2. Foundations of Euclid's geometry
3. The Hilbert axioms: incidence and order
4. Congruence, continuity and the parallel axiom
5. Absolute geometry
6. Affine geometry over a field
7. Non Euclidean geometries
8. The projective viewpoint
9. Projective axioms
10. Affine space and projective space

Elements of projective geometry
11. The projective space of a vector space
12. Homogeneous coordinates and Grassmann formula
13. The Fano and Pappos configurations
14. Desargues theorem and the theorem of coordination
15. Cross ratio and the fundamental theorem of projective geometry
16. Plücker coordinates and epipolar geometry

Affine geometry
17. The affine space on a vector space
18. Subvarieties and Grassmann formulas
19. Coordinates and equations
20. Affine maps
21. Some interesting affine maps
22. Two important theorems in affine geometry
23. Euclidean affine space
24. Rigid motions
25. Classification of rigid motions

Quadrics
26. Quadrics
27. Four points of view on quadrics
28. Classification theorems
29. Projective classification over the real field, the complex field and the finite fields
30. Affine classification of quadrics

Methodology

There will be 30 lessons plus 15 problem solving sessions. Also, there will be 4 seminar activities.

Activities

Title	Hours	ECTS	Learning Outcomes
Type: Directed			
exercises	15	0.6	
lessons	30	1.2	
seminars	8	0.32	
Type: Autonomous			
problem solving	41	1.64	
study	30	1.2	
test oriented study	10	0.4	

Assessment

The final grade will be obtained from the grades of the two tests plus the grade obtanined in the seminar activities, according to the indicated weights.

Assessment Activities

Title	Weighting	Hours	ECTS	Learning Outcomes
Second chance test	60%	4	0.16	1, 2, 4, 3
Seminar sessions	40%	4	0.16	6, 5
Test #1	30%	4	0.16	3
Test #2	30%	4	0.16	1, 2, 4, 3

Bibliography

Euclides, *"Elements de Geometria"*

D. Hilbert, *"Grundlagen der Geometrie"*

R. Hartshorne, *"Geometry: Euclid and beyond"*

A. Reventós, *"Geometria Projectiva"*

A. Reventós, *"Afinitats, moviments i quàdriques"*

J. Kock, *"Lliçons de geometria afí."* [http://mat.uab.cat/~kock/docencia/GL/]

J. Aguadé, "*Un curs de geometria lineal*"